

# MCFM-9.0 manual

## A Monte Carlo for FeMtobarn processes at Hadron Colliders

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## 1. Overview

MCFM is a parton-level Monte Carlo program that gives NLO and NNLO predictions for a wide range of processes at hadron colliders. The program has been developed over a number of years and results have been presented in a number of published papers. The papers describing the original code and the most significant developments in the NLO implementation are:

- J. M. Campbell and R. K. Ellis,  
“*An update on vector boson pair production at hadron colliders,*”  
Phys. Rev. D **60**, 113006 (1999) [arXiv:hep-ph/9905386].
- J. M. Campbell, R. K. Ellis and C. Williams,  
“*Vector boson pair production at the LHC,*”  
JHEP **1107**, 018 (2011) [arXiv:1105.0020 [hep-ph]].
- J. M. Campbell, R. K. Ellis and W. Giele,  
“*A Multi-Threaded Version of MCFM,*”  
EPJ **C75**, 246 (2015) [arXiv:1503.06182 [hep-ph]].

As of v8.0 MCFM can also compute selected color-singlet processes through NNLO in QCD perturbation theory. The processes available at this precision, as well as benchmark numbers, are detailed in Section 5. When using MCFM 8.0 or newer for NNLO calculations please refer to:

- R. Boughezal, J. M. Campbell, R. K. Ellis,  
C. Focke, W. Giele, X. Liu, F. Petriello and C. Williams,  
“*Color singlet production at NNLO in MCFM,*” arXiv:1605.08011.

A significant overhaul of MCFM was undertaken in v9.0 of MCFM. If you use this version or newer please cite

- John Campbell and Tobias Neumann. “Precision phenomenology with MCFM” (2019).  
arXiv: 1909.09117 [hep-ph]

Other relevant references, corresponding to publications associated with the implementation of specific processes at NLO and NNLO, are listed in appendix B.

## 2. Installation

The MCFM package may be downloaded from the MCFM homepage at <https://mcfm.fnal.gov>. After extracting, the source can be compiled by first running the `./Install` command, which compiles some tensor reduction components as well as the included libraries QCDLoop and QD. Then the main part can be compiled with `make` (we require GNU make). By default, the Fortran and C compilers from GCC are used, but we also allow for the Intel Fortran and C compilers. These can be enabled by setting the flag `USEINTEL` in the `Install` and `makefile` files. For the GNU compiler collection at least version 7 is required.

If your GNU Fortran and GNU C++ compiler commands are different than `gfortran` and `g++` you should export the environment variables `FC` and `CXX` to match the names of the `gfortran` and `g++` compiler commands before running the `Install` script or the `makefile`. These variables can also be adjusted in the `Install` and `makefile` files.

Additional complications may arise on OS X systems, where by default gcc/g++ is linked to the clang compiler. Please make sure that the g++ command used in the files Install and makefile correspond to the actual GNU g++ compiler command, or export the FC and CXX environment variables.

To enable the use of LHAPDF, the variable PDFROUTINES in the makefile should be set to LHAPDF, and the path LHAPDFLIB can be adjusted to the path containing libLHAPDF.so. This version of MCFM has been explicitly tested against LHAPDF-6.2.1. LHAPDF version 5 is not supported.

If at the first execution of MCFM the library libLHAPDF cannot be found, you can add your LHAPDF library path to the environment variable LD\_LIBRARY\_PATH as follows:

```
export LD_LIBRARY_PATH=${LD_LIBRARY_PATH}:/home/user/newpath/lib
```

Please ensure that your compiler is working and can produce executable program files. For example when your compiler has been installed into a non-standard location you probably need to append the compiler library path to LD\_LIBRARY\_PATH (DYLD\_FALLBACK\_LIBRARY\_PATH on OS X). This can be achieved, for example, as follows:

```
export LD_LIBRARY_PATH=${LD_LIBRARY_PATH}:/home/user/local/lib/gcc7
```

The directory structure of MCFM is as follows:

- **Doc.** The source for this document.
- **Bin.** The directory containing the executable mcfm\_omp, and various essential files – notably the options file input.ini.
- **Bin/Pdfdata.** The directory containing the internal PDF data-files.
- **obj.** The object files produced by the compiler.
- **src.** The Fortran source files in various subdirectories.
- **qcdloop-2.0.2.** The source files to the library QCDLoop [2, 3].
- **TensorReduction** General tensor reduction code based on the work of Passarino and Veltman [4] and Oldenborgh and Vermaseren [5].
- **qd-2.3.22.** Library to support double-double and quad-double precision data types [6].

## 2.1. OpenMP and MPI

MCFM uses OMP (Open Multi-Processing) to implement multi-threading and automatically adjusts to the number of available CPU threads. The multi-threading is implemented with respect to the integration routine Vegas, which distributes the event evaluations over the threads and combines all events at the end of every iteration.

Two environment variables are useful. On some systems, depending on the OMP implementation, the program will crash when calculating some of the more complicated processes, for example  $W + 2$  jet production at NLO. Then, adjusting `OMP_STACKSIZE` may be needed for the program to run correctly. Setting this variable to 16000, for instance in the Bash shell by using the command `export OMP_STACKSIZE=16000`, has been found to be sufficient for all processes. The second useful variable `OMP_NUM_THREADS` may be used to directly control the number of threads used during OMP execution (the default is the maximum number of threads available on the system).

It is also possible to run MCFM using MPI (Message Passing Interface). To run in this mode, change the flag `USEMPI` in the makefile to `YES` and specify the MPI compiler wrappers and compilers in the makefile or set the environment variables `FC` and `CXX`. By default, the OpenMPI compiler wrappers `mpifort` and `mpic++` are used, to use `gfortran` and `g++`. When `USEINTEL` is set, `mpiifort` and `mpicc` are used.

## 3. Description of new features in MCFM-9.0

In this section we present the new and modified features in MCFM-9.0 and describe how to use them on a technical level. This serves mostly as a quick-start for users already familiar with MCFM. With all re-implemented and newly implemented components we strive for Fortran 2008 compliance, making explicit use of its features. Following the Fortran standard furthermore allows us to achieve compatibility with not just the GNU compiler. In previous versions of MCFM the licensing was unclear, since none was specified. We now license all code under the GNU GPL 3 license<sup>1</sup>. For supporting material we recommend studying the release paper of MCFM-9.0 in ref. [1], from which this section is taken.

**Improved input file mechanism.** We have implemented a new input file mechanism based on the configuration file parser `config_fortran` [7]. This INI-like file format no longer depends on a strict ordering of configuration elements, allows easy access to configuration elements through a single global configuration object, and makes it easy to add new configuration options of scalar and array numerical and string types. Using the parser package also allows one to override or specify all configuration options as command line arguments to MCFM, for example running MCFM like `./mcfm_omp input.ini -general%nproc=200 -general%part=nlo`. This is useful for batch parameter run scripts. Settings can also be overridden with additional input files that specify just a subset of options.

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<sup>1</sup>See <https://www.gnu.org/licenses/gpl-3.0.en.html>.

**New histogramming.** We replaced the previous Fortran77 implementation of histograms, that used routines from 1988 by M. Mangano, with a new suite of routines. The new histogram implementation allows for any number of histograms with any number of bins, each of which is dynamically allocated. Furthermore, everything is also handled in a fully multi-threaded approach with the integration. For each OMP thread temporary histograms are allocated which are then reduced to a single one after each integration iteration, so that no OMP locks (critical regions) are required.

**New Vegas integration, part-adaptive and resumable.** The previous implementation of the Vegas routine was based on Numerical Recipes code. We have re-implemented Vegas and the surrounding integration routines. All parts of a NLO or NNLO calculation are now chosen adaptively based on the largest absolute numerical uncertainty. A precision goal can be set in the input file as well as a  $\chi^2/\text{it}$  goal and a precision goal for the warmup run. If the goals for the warmup are not reached, the warmup repeats with twice the number of calls. With the setting `writeintermediate` one can control whether histograms are written in intermediate stages during the integration. Enabling the setting `readin` allows one to resume the integration from any point from a previous run. Snapshots saving the whole integration state are saved automatically. When resuming, the only parameter that the user can safely officially change is the `precisiongoal`. Further tweak configuration options to control the stages of the integration have been introduced, which can provide benefits over the default settings in certain situations.

The section `integration` in the configuration file allows for tweaks in the following way. The precision goal can be adjusted by setting `precisiongoal` to a relative precision that should be reached. Similarly, the settings `warmupprecisiongoal` and `warmupchisqgoal` control the minimum relative precision and  $\chi^2/\text{it}$  for the warmup phase of `iterbatchwarmup` (default 5) iterations. If the warmup criterion fails, the number of calls is increased by a factor of two. The calls per iteration get increased by a factor of `callboost` (default 4) after the warmup. From then on the number of calls per iteration is increased by a factor of `itercallmult` (default 1.4) for a total of `iterbatch1` iterations. After these first `iterbatch1` iterations, the increase happens for every `iterbatch2` iterations. The setting `maxcallsperiter` controls the cap for the number of calls per iteration. The number of Vegas grid subdivisions can be controlled with `ndmx` (default 100).

The purpose of these settings is a fine control in certain situations. For example to compute expensive PDF uncertainties, one wants a relatively precise warmup run (where additional PDF sets are not sampled) and as few calls as necessary afterwards: For the plots in this paper we thus chose a relative warmup precision goal of 10%, and set `callboost` to 0.25. This means that the first `iterbatch1` iterations after the warmup run only with a quarter of the calls than during the warmup. This precision is sufficient to compute precise PDF uncertainties, when making use of the strong correlations as in MCFM-9.0. Any further iterations come in batches of `iterbatch2`, which we set to 1. It allows for a quick switching to parts of the NNLO cross section that have the largest uncertainty. For normal applications one wants to boost the number of calls after the warmup significantly, so a default value of `callboost=4` is chosen.

We provide default settings for the initial number of calls per iteration for all components of a NNLO calculation. They can be overridden with the following settings in the `integration` section: `initcallslord`, `initcallsnlovirt`, `initcallsnloreal`, `initcallsnlofrag` for parts

of a NLO calculations, `initcallssnlobelow`, `initcallssnloabove` for parts of a SCET based NLO calculation, and `initcallsnnlobelow`, `initcallsnnlovirtabove`, as well as `initcallsnnlorealabove` for the parts of the NNLO coefficient.

**Low discrepancy sequence.** MCFM-8.0 and prior relied on a linear congruential generator implementation from Numerical Recipes for the generation of a pseudo-random sequence. With newer versions the MT19937 implementation of the C++ standard library is used, and with this version of MCFM we include an implementation of the Sobol low discrepancy sequence based on the code `sobseq` [8] with initialization numbers from ref. [9]. The Sobol sequence is used by default and can be toggled using the flag `usesobol = .true.` in the `integration` section of the input file, see ref. [1]. When running in MPI mode, the number of nodes has to be a power of two for the Sobol sequence, because we use it in a strided manner. Otherwise the code will automatically fall back to using the MT19937 sequence with seed value `seed` in the integration section of the input file. A `seed` value of 0 denotes a randomly initialized seed.

**Fully parallelized OMP+MPI use of LHAPDF.** In previous versions of MCFM calls to LHAPDF were forced to access from only a single OMP thread through a lock. This is because the interface was based on the old LHAglue interface, part of LHAPDF. We have written an interface to LHAPDF from scratch based on the new object oriented treatment of PDFs in LHAPDF 6. For each OMP thread we initialize a copy of the used PDF members which can be called fully concurrently. The amount of PDF sets with or without PDF uncertainties is only limited by the available system memory. The memory usage of MCFM can then range from roughly 20MB when only one central PDF grid is being used, to  $\sim 7.4$  GB when 32 OMP threads fully load all members of the PDF sets `CT14nnlo`, `MMHT2014nnlo68c1`, `ABMP16als118_5_nnlo`, `NNPDF30_nnlo_as_0118`, `NNPDF31_nnlo_as_0118` and `PDF4LHC15_nnlo_30` for PDF uncertainties. The total number of members for these grids is 371, each loaded for every of the 32 OMP threads.

Since each OMP thread allocates its own copy of PDF members and histograms we have no need to introduce any OMP locks. On the other hand the memory usage increases and one runs into being CPU cache or DRAM bandwidth bound earlier. In practice, we find that this is still faster than having OMP locks, which directly decrease the speedup in the spirit of Amdahl's law. Ideally the LHAPDF library should be improved to allow for thread-safe calls with just one memory allocation.

**Histograms for additional values of  $\tau_{\text{cut}}$ ,  $\mu_R$ ,  $\mu_F$  and multiple PDFs.** When using the automatic scale variation, in addition to the normal histograms, additional histograms with filenames `_scale_XY_` are generated, where X is a placeholder for the renormalization scale variation and Y for the factorization scale variation. X and Y can either be u for an upwards variation by a factor of two, d for a downwards variation by a factor of two, or just - if no change of that scale was made. The envelope of maximum and minimum can then easily be obtained.

For the sampling of additional values of  $\tau_{\text{cut}}$  for NLO and NNLO calculations using jettiness subtractions, additional histograms with filenames `_taucut_XXX_` are written. Here XXX is a placeholder for the chosen  $\tau_{\text{cut}}$  values in the optional array `taucutarray`, if specified, or one of



the five automatically chosen values. These additional files only contain the *differences* to the nominal choice of  $\tau_{\text{cut}}$ , so that  $\Delta\sigma(\tau_{\text{cut,nominal}}) - \Delta\sigma(\tau_{\text{cut,i}})$  is stored. If `taucutarray` has not been specified, the automatic choice of additional  $\tau_{\text{cut}}$  values is enabled based on the default nominal  $\tau_{\text{cut}}$  for the process or the users choice of the nominal  $\tau_{\text{cut}}$  value as specified in `taucut`. In addition a file with `_taucutfit_` is generated, which in addition to the fitted corrections and its uncertainty includes columns for the maximum relative integration uncertainty for the additionally sampled  $\tau_{\text{cut}}$  values and the reduced  $\chi^2$  of the fit. With the procedure in section 5, the fit together with the individual  $\tau_{\text{cut}}$  histograms allows the user to assess the systematic  $\tau_{\text{cut}}$  error and possibly improve results.

When multiple PDF sets are chosen, additional files with the names of the PDF sets are generated. In case PDF uncertainties are enabled, the histograms also include the upper and lower bounds of the PDF uncertainties.

**User cuts, histograms and re-weighting.** Modifying the plotting routines in the files `src/User/nplotter*.f` allows for modification of the pre-defined histograms and addition of any number of arbitrary observables. The routine `gencuts_user` can be adjusted in the file `src/User/gencuts_user.f90` for additional cuts after the jet algorithm has performed the clustering. In the same file the routine `reweight_user` can be modified to include a manual re-weighting for all integral contributions. This can be used to obtain improved uncertainties in, for example, tails of distributions. One example is included in the subdirectory `examples`, where the `reweight_user` function approximately flattens the Higgs transverse momentum distribution, leading to equal relative uncertainties even in the tail at 1 TeV.

### 3.1. Compatibility with the Intel compiler and benchmarks

Previous versions of MCFM were developed using `gfortran` as a compiler. MCFM contained code that did not follow a specific Fortran standard, and was only compatible with using `gfortran`. We fixed code that did not compile or work with the recent Intel Fortran compiler `ifort 19.0.1`. This does not mean that we claim to be strictly standards compliant with a specific Fortran version, but we aim to be compliant with Fortran 2008. We now fully support GCC versions newer than 7 and Intel compilers newer than 19. There might still be compatibility issues with other Fortran compilers, but we are happy to receive bug reports for any issues regarding compilation, that are not due to a lack of modern Fortran 2008 features. To use the Intel compiler one has to change the `USEINTEL` flag in the files `Install` and `makefile` to `YES`.

To see whether MCFM can make use of potential Intel compiler improvements over the GNU compiler collection (GCC) we benchmarked the double real emission component of Higgs production at NNLO. We perform tests on our cluster with Intel Xeon 64-bit X5650 2.67 GHz Westmere CPUs, where two six-core CPUs are run in a dual-socket mode with a total of twelve cores. Similarly, we have an AMD 6128 HE Opteron 2GHz quad-socket eight-core setup, thus each having 32 cores per node.

We benchmark both the Intel and GCC compilers on both the Intel and AMD systems. On the Intel system we use 16 MPI processes each with 12 OMP threads, and on the AMD system we have 8 MPI processes using 32 OMP threads. With this we have the same total clockrate

Table 1: Benchmark results on the Intel system with  $10 \cdot 25\text{M}$  calls distributed over 16 MPI processes, each using 12 OMP threads. The GCC version is 9.1.0 and the Intel Fortran compiler 19.0.1

Compiler/flags	wall time $\pm 0.5\text{s}$
ifort -O3 -xsse4.2	90s
ifort -O2 -xsse4.2	86s
ifort -O2	90s
ifort -O1	103s
gfortran -O3 -march=westmere	101s
gfortran -O2 -march=westmere	105s
gfortran -O2	105s
gfortran -O1	110s

of 512 GHz for each setup. For all benchmarks we find that the scaling is perfect up to this size, that is if we use half the number of MPI or OMP threads we double our run-time.

We first try both the Intel fortran compiler 19.0.1 and GCC 9.1.0 on the Intel system with the highest generic optimization flags `-O3 -xsse4.2` and `-O3 -march=westmere`, respectively. Furthermore, we lower the optimizations to `-O2` each and remove the processor specific optimization flags `-xsse4.2` and `-march=westmere`, respectively. All our benchmark run-times in the following are consistent within  $\pm 0.5\text{s}$ .

We do not support enabling unsafe math operations with `-ffast-math`, since the code relies on the knowledge of NaN values and checks on those. Such checks would be skipped with the meta flag `-ffast-math` which sets `-ffinite-math-only`.

The benchmark results in table 1 show that using the Intel compiler, performance benefits of  $\simeq 10 - 20\%$  can be achieved. Our goal here is not to go beyond this and check whether exactly equivalent optimization flags have been used in both cases. Enabling optimizations beyond `-O2` have little impact, but come with a penalty for the Intel compiler and with a slight benefit for gfortran. We also notice that processor specific optimizations play no significant role. This might also be in part due to the fact that MCFM does not offer much space for (automatic) vectorization optimizations. To summarize, the default optimization flags of `-O2` should be sufficient in most cases. We do not expect that the conclusions from these benchmarks change for different processes. On the other hand if computing PDF uncertainties, the majority of time is used by LHAPDF and different optimization flags for LHAPDF might play a role then. We performed the same benchmark with an older version of GCC, version 7.1.0 using `-O2` optimizations, and found that the run-times are the same as for the newer version.

Finally, we performed some benchmarks on our AMD setup and found that it is  $\simeq 2.5$  times slower for the same total clockrate. Using the Intel compiler for the AMD setup decreased the performance by another  $\simeq 30\%$ . This is likely due to the fact that the Intel compiler already optimizes for the general Intel architecture.

These benchmarks try to give a general impression and might depend in detail on the process, the number of histograms and whether to compute PDF uncertainties, for example. Especially when computing PDF uncertainties the perfect scaling we tested here might break down

since the computation can become memory bound. We discuss this caveat in more detail in section 3.2.

### 3.2. Remarks on memory bound performance issues

To get numerically precise predictions at the per mille level for NNLO cross sections, already hundreds of million of calls are necessary. Obtaining PDF uncertainties using those NNLO matrix elements significantly increases the computational time. In a simplified view the total computational time composes as  $N_{\text{calls}} * (T + N_{\text{PDF}} \cdot T_{\text{PDF}})$ , where  $T$  is the computational effort for the matrix element piece, and the PDF part is proportional to the time calling the PDF evolution  $N_{\text{PDF}}$  times and code related to performing the convolutions. For tree level matrix element evaluations, usually also  $T \lll T_{\text{PDF}}$  holds, so the computational cost grows linearly with the number of PDFs.

This naive picture breaks down in practice when a lot of PDFs are sampled together with a lot of histograms or histogram bins. The total memory necessary to store all the histogram information grows like  $N_{\text{PDF}} \cdot N_{\text{bins}} \cdot N_{\text{thr.}}$ , where  $N_{\text{PDF}}$  is the number of PDF members,  $N_{\text{bins}}$  the number of histogram bins summed over all histograms and  $N_{\text{thr.}}$  is the number of OMP threads. The factor  $N_{\text{thr.}}$  enters since we have thread-local storage to avoid OMP locks. The values are stored in double precision, so the total memory used is  $N_{\text{PDF}} \cdot N_{\text{bins}} \cdot N_{\text{thr.}} \cdot 8$  bytes.

Assuming for example, 300 PDF members, 10 histograms with each 20 bins and 12 threads, this sums up to 720 kb of memory. For the virtual corrections and LO pieces, one has to update this amount of memory once for each call. For the real emission matrix elements one has to accumulate all dipole contributions, so this number additionally scales with the number of dipole contributions. All the histogram updates are usually fully vectorized for modern superscalar processors with SSE and/or AVX extensions. But if this used memory is too large and does not easily fit into the CPU core caches anymore, a transfer to and from DRAM happens, which now is the limiting factor and significantly slows down the computation. Because for that reason, one should work with a minimal number of necessary histograms when working with a lot of PDF members. This is especially important for cluster setups that are not optimized towards memory bound applications, non-NUMA systems. For example in our cluster we have relatively old AMD Opteron quad-socket eight-core nodes with little CPU cache, and with above numbers we are already limited in wall-time improvements with using  $\sim 16$  cores. Then reducing the number of histograms will *significantly* improve the performance. In principle one can reduce the histogram precision to single precision and cut memory transfer and storage in half, while doubling the computational speed. This might lead to problems with accumulated rounding errors though, and we have not investigated this further, since in practice one can sufficiently limit the number of histograms or PDF sets.

## 4. Configuration

### 4.1. Compile-time settings

MCFM allows the user to choose between a number of schemes for defining the electroweak couplings. These choices are summarized in Table 2. The scheme is selected by modifying the

Table 2: Different options for the scheme used to fix the electroweak parameters of the Standard Model and the corresponding default input values.  $M_W$  and  $M_Z$  are taken from ref. [10].

Parameter	Name ( <code>_inp</code> )	Input Value	Output Value determined by <code>ewscheme</code>			
			-1	0	1	2
$G_F$	<code>Gf</code>	$1.16639 \times 10^{-5}$	input	calculated	input	input
$\alpha(M_Z)$	<code>aemmz</code>	1/128.89	input	input	calculated	input
$\sin^2 \theta_w$	<code>xw</code>	0.2223	calculated	input	calculated	input
$M_W$	<code>wmass</code>	80.385 GeV	input	calculated	input	calculated
$M_Z$	<code>zmass</code>	91.1876 GeV	input	input	input	calculated
$m_t$	<code>mt</code>	<code>input.ini</code>	calculated	input	input	input

Table 3: Default values for the remaining parameters in MCFM.  $\Gamma_W$  and  $\Gamma_Z$  from ref. [10].

Parameter	Fortran name	Default value
$m_\tau$	<code>mtau</code>	1.777 GeV
$m_\tau^2$	<code>mtausq</code>	3.1577 GeV <sup>2</sup>
$\Gamma_\tau$	<code>tauwidth</code>	$2.269 \times 10^{-12}$ GeV
$\Gamma_W$	<code>wwidth</code>	2.093 GeV
$\Gamma_Z$	<code>zwidth</code>	2.4952 GeV
$V_{ud}$	<code>Vud</code>	0.975
$V_{us}$	<code>Vus</code>	0.222
$V_{ub}$	<code>Vub</code>	0.
$V_{cd}$	<code>Vcd</code>	0.222
$V_{cs}$	<code>Vcs</code>	0.975
$V_{cb}$	<code>Vcb</code>	0.

value of `ewscheme` in `src/User/mdata.f` prior to compilation, which also contains the values of all input parameters (see also Table 3).

The default scheme corresponds to `ewscheme=+1`. As described below, this corresponds to a scheme in which the top quark mass is an input parameter so that it is more suitable for many processes now included in the program.

The choice of (`ewscheme=-1`) enforces the use of an effective field theory approach, which is valid for scales below the top mass. In this approach there are 4 independent parameters (which we choose to be  $G_F$ ,  $\alpha(M_Z)$ ,  $M_W$  and  $M_Z$ ). For further details, see Georgi [11].

For all the other schemes (`ewscheme=0,1,2`) the top mass is simply an additional input parameter and there are 3 other independent parameters from the remaining 5. The variable `ewscheme` then performs exactly the same role as `idef` in MadEvent [12]. `ewscheme=0` is the old MadEvent default and `ewscheme=1` is the new MadEvent default, which is also the same as that used in Alpgen [13] and LUSIFER [14] For processes in which the top quark is directly produced it is preferable to use the schemes (`ewscheme=0,1,2`), since in these schemes one can adjust the top mass to its physical value (in the input file `input.ini`).

## 4.2. Parton distributions

The value of  $\alpha_s(M_Z)$  is not adjustable; it is hardwired with the parton distribution. In addition, the parton distribution also specifies the number of loops that should be used in the running of  $\alpha_s$ . The default mode of operation is to choose from a collection of modern parton distribution functions that are included with MCFM. The distributions, together with their associated  $\alpha_s(M_Z)$  values, are given in table 4 on page 13. In addition to these choices, a number of historical PDF sets are also available; for details see the manual for MCFM-8.0. Note that, due to the memory requirements for using the NNPDF sets, in OpenMP operation it is usually necessary to increase the value of the environment variable `OMP_STACKSIZE` to avoid segmentation faults.

pdlabel	$\alpha_s(M_Z)$	order	reference
mstw8lo	0.1394	1	[15]
mstw8nl	0.1202	2	[15]
mstw8nn	0.1171	3	[15]
MMHT_lo	0.135	1	[16]
MMHT_nl	0.120	2	[16]
MMHT_nn	0.118	3	[16]
CT10.00	0.118	2	[17]
CT14.LL	0.130	1	[18]
CT14.NL	0.118	2	[18]
CT14.NN	0.118	3	[18]
CT14qed	0.118	2	[19]
NN2.3NL	0.118	2	[20]
NN2.3NN	0.118	3	[20]
NN3.0LO	0.118	1	[21]
NN3.0NL	0.118	2	[21]
NN3.0NN	0.118	3	[21]

Table 4: Modern PDF sets that are available in the code, their corresponding values of  $\alpha_s(M_Z)$  and order of running, and a reference to the paper that describes their origin. Further sets, of a more historical nature, are listed in appendix A.

## 4.3. Electroweak corrections

As of version 8.1, MCFM allows the calculation of weak corrections to a selection of processes: 31 (neutral-current DY), 157 (top-pair production) and 190 (di-jet production). This is controlled by the flag `ewcorr` in the input file. A complete description of the calculations is provided in Ref. [22].

By setting `ewcorr` to `sudakov`, the program performs a calculation of the leading weak corrections to these processes using a Sudakov approximation that is appropriate at high energies. The calculation of the weak corrections using the exact form of the one-loop amplitudes is obtained by using the flag `exact`. A comparison between the two approaches,

together with discussions of the validity of the Sudakov approximation, may be found in Ref. [22].

For the case of top-pair and di-jet production, the weak one-loop corrections contain infrared divergences that must be cancelled against corresponding real radiation contributions (in much the same manner as a regular NLO QCD calculation). For this reason the screen output will contain two sets of iterations corresponding to the virtual and real contributions.

For all processes, performing the calculation of weak corrections enables a special mode of phase-space integration that is designed to better-sample events produced at high-energies. For this reason the VEGAS output that appears on the screen does not correspond to a physical cross-section – and a corresponding warning message to this effect will be displayed. In many cases the quantity of most interest is the relative correction to the leading order result ( $\delta_{\text{wk}}$ ) given by,

$$\delta_{\text{wk}} = \frac{d\sigma_{\text{wk}}^{\text{NLO}} - d\sigma^{\text{LO}}}{d\sigma^{\text{LO}}} . \quad (1)$$

It is straightforward to compute this quantity for a distribution by editing the appropriate `nplotter` routine. This is achieved by filling a histogram with the weight corresponding to the LO result, another with the weight for the NLO weak result and then an additional placeholder histogram that contains the special string '+RELEW+'. Examples of the syntax and correct calling sequence can be seen in the code.

#### 4.4. Nuclear collisions

It is possible to specify nuclear collisions by choosing values of `ih1` and/or `ih2` above 1000d0. In that case, the identity of the nucleus is specified by the atomic number and mass ( $Z$  and  $A$  respectively) as follows:

$$ih = 1000Z + A. \quad (2)$$

For example, to choose an incoming lead beam one would set `ih1=+82207d0`, corresponding to  $Z = 82$  and  $A = 207$ . When running the program, the value of `sqrts` should also be changed. This must be done by hand and is not automatically taken care of by the program. The centre-of-mass energy is decreased by a factor of  $\sqrt{Z/A}$  for each nuclear beam.

The nucleon PDFs are calculated by applying the correction factors of EKS98 [23] on top of the PDF set that is selected. This construction simply corrects each parton distribution by a factor that depends on the value of  $(x, \mu)$  in the event. This parametrization is limited to the region  $\mu < 100$  GeV and any value above that threshold will instead default to 100 GeV.

Note that the cross-section reported by the program at the end of the run is given per nucleon per beam. Therefore the appropriate factors of  $A$  should be applied in order to obtain the total cross section.

#### 4.5. Run-time input file configuration

MCFM execution is performed in the `Bin/` directory, with syntax:

```
mcfm_omp input.ini
```

If no command line options are given, then MCFM will default to using the file `input.ini` in the current directory for choosing options. The `input.ini` file can be in any directory and then the first argument to `mcfm_omp` should be the location of the file. Furthermore, one can overwrite or append single configuration options with additional parameters like `./mcfm_omp benchmark/input.ini -general%part=nlo -lhpdf%dopdferrors=.true..` Here specifying a parameter uses a single dash, then the section name as in the configuration file, followed by a percent sign, followed by the option name, followed by an equal sign and the actual value of the setting.

All default settings in the input file are explained below, as well as further optional parameters. The top level setting `mcfm_version` specifies the input file version number and it must match the version of the code being used.

Section general	Description
nproc	The process to be studied is given by choosing a process number, according to Table 33 in Appendix C. $f(p_i)$ denotes a generic partonic jet. Processes denoted as “LO” may only be calculated in the Born approximation. For photon processes, “NLO+F” signifies that the calculation may be performed both at NLO and also including the effects of photon fragmentation and experimental isolation. In contrast, “NLO” for a process involving photons means that no fragmentation contributions are included and isolation is performed according to the procedure of Frixione [24].

<code>part</code>	<p>This parameter has 5 possible values, described below:</p> <ul style="list-style-type: none"> <li>• <code>lo</code> (or <code>lord</code>). The calculation is performed at leading order only.</li> <li>• <code>virt</code>. Virtual (loop) contributions to the next-to-leading order result are calculated (+counterterms to make them finite), including also the lowest order contribution.</li> <li>• <code>real</code>. In addition to the loop diagrams calculated by <code>virt</code>, the full next-to-leading order results must include contributions from diagrams involving real gluon emission (-counterterms to make them finite). Note that only the sum of the <code>real</code> and the <code>virt</code> contributions is physical.</li> <li>• <code>nlo</code> (or <code>tota</code>). For simplicity, the <code>nlo</code> option simply runs the <code>virt</code> and <code>real</code> real pieces in series before performing a sum to obtain the full next-to-leading order result. For photon processes that include fragmentation, <code>nlo</code> also includes the calculation of the fragmentation (<code>frag</code>) contributions.</li> <li>• <code>nlocoeff</code>. This computes only the contribution of the NLO coefficient; it is equivalent to running <code>nlo</code> and then subtracting the result of <code>lo</code>.</li> <li>• <code>nlodk</code> (or <code>todk</code>). Processes 114, 161, 166, 171, 176, 181, 186, 141, 146, 149, 233, 238, 501, 511 only, see sections 6.36 and 6.38 below.</li> <li>• <code>frag</code>. Processes 280, 285, 290, 295, 300-302, 305-307, 820-823 only, see sections 6.58, 6.61 and 6.62 below.</li> <li>• <code>nnlo</code> (and <code>nnlocoeff</code>). The computation of the NNLO prediction (or the NNLO coefficient in the expansion) is described separately below.</li> </ul>
<code>runstring</code>	When MCFM is run, it will write output to several files. The label <code>runstring</code> will be included in the names of these files.
<code>sqrts</code>	Center of mass energy in GeV.
<code>ih1, ih2</code>	The identities of the incoming hadrons may be set with these parameters, allowing simulations for both $p\bar{p}$ (such as the Tevatron) and $pp$ (such as the LHC). Setting <code>ih1</code> equal to <code>+1</code> corresponds to a proton, whilst <code>-1</code> corresponds to an anti-proton. Values greater than <code>1000d0</code> represent a nuclear collision, as described in Section 4.4.
<code>zerowidth</code>	When set to <code>.true.</code> then all vector bosons are produced on-shell. This is appropriate for calculations of <i>total</i> cross-sections (such as when using <code>removebr</code> equal to <code>.true.</code> , below). When interested in decay products of the bosons this should be set to <code>.false.</code>
<code>removebr</code>	When set to <code>.true.</code> the branching ratios are removed for unstable particles such as vector bosons or top quarks. See the process notes in Section 6 below for further details.



<code>ewcorr</code>	Specifies whether or not to compute EW corrections for the process. Default is <code>none</code> . May be set to <code>exact</code> or <code>sudakov</code> for processes 31 (neutral-current DY), 157 (top-pair production) and 190 (di-jet production). For more details see section 4.3.
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Section <code>nnlo</code>	Description
<code>taucut</code>	Optional. This sets the value of the jettiness variable $\tau_{\text{cut}}$ , as multiplied by the invariant mass of the Born system, that separates the resolved and unresolved regions in NNLO calculations that use zero-jettiness. The default value results in total inclusive cross sections with less than 1% residual cutoff effects, see section 5.
<code>tcutarray</code>	Optional. Array that specifies multiple <code>taucut</code> values that should be sampled on the fly in addition to the nominal <code>taucut</code> value. Both larger and smaller values than the nominal one can be specified, although uncertainties for smaller values will be large. We generally do not recommend smaller values than the nominal one chosen with <code>taucut</code> . Default values are chosen to be 2, 4, 8, 20, 40 times the nominal choice of <code>taucut</code> .
<code>dynamictau</code>	Optional. If <code>.false.</code> , the <code>taucut</code> value specified is not multiplied by the invariant mass of the Born system. Default is <code>.true.</code>

---

Section <code>pdf</code>	Description
<code>pdlabel</code>	This specifies the parton distributions used in case the code has been built with <code>PDFROUTINES = NATIVE</code> . The choice of parton distribution is made by inserting the appropriate 7-character code from the table in section 4.2 or in appendix A for historical PDF sets. As mentioned above, this also sets the value of $\alpha_S(M_Z)$ .

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Section <code>lhpdf</code>	Description
<code>lhpdfset</code>	Specifies the parton distributions used in case the code has been built with <code>PDFROUTINES = LHAPDF</code> . For a default global installation the PDFs reside in <code>/usr/share/LHAPDF/</code> or <code>/usr/local/share/LHAPDF/</code> , and the name equals the set name from <a href="https://lhpdf.hepforge.org/pdfsets.html">https://lhpdf.hepforge.org/pdfsets.html</a> , which is also the directory name of the sets. Multiple PDF sets separated by a space can be specified.
<code>lhpdfmember</code>	Specifies the individual members of the parton distribution sets. A value of zero corresponds to the central value for Hessian sets. In case multiple sets have been specified above, each one needs a member number separated by space.
<code>dopdferrors</code>	When this is set to <code>.true.</code> PDF uncertainties are calculated for every specified PDF set according to the routines provided by LHAPDF. The <code>lhpdfmember</code> numbers are ignored but must still be set for each member.

---

Section scales	Description
<code>renscale</code>	This parameter may be used to adjust the value of the <i>renormalization</i> scale. This is the scale at which $\alpha_S$ is evaluated and will typically be set to a mass scale appropriate to the process ( $M_W$ , $M_Z$ , $M_t$ for instance).
<code>facscale</code>	This parameter may be used to adjust the value of the <i>factorization</i> scale and will typically be set to a mass scale appropriate to the process ( $M_W$ , $M_Z$ , $M_t$ for instance).
<code>dynamicscale</code>	This character string is used to specify whether the renormalization, factorization and fragmentation scales are dynamic, i.e. recalculated on an event-by-event basis. If this string is set to ‘ <b>none</b> ’ then the scales are fixed for all events at the values specified by <code>renscale</code> , <code>facscale</code> as well as <code>fragmentation_scale</code> as defined further below. The type of dynamic scale to be used is selected by using a particular string for the variable <code>dynamicscale</code> , as indicated in table 11 on page 20. Not all scales are defined for each process, with program execution halted if an invalid selection is made in the input file. The selection chooses a reference scale, $\mu_0$ . The actual scales used in the code are then,
	$\mu_{\text{ren}} = \text{scale} \times \mu_0, \quad \mu_{\text{fac}} = \text{facscale} \times \mu_0 \quad (3)$
	Note that, for simplicity, the fragmentation scale (relevant only for processes involving photons) is set equal to the renormalization scale. In some cases it is possible for the dynamic scale to become very large. This can cause problems with the interpolation of data tables for the PDFs and fragmentation functions. As a result if a dynamic scale exceeds a maximum of 60 TeV (PDF) or 990 GeV (fragmentation) this value is set by default to the maximum.
<code>doscalevar</code>	This additional option can be set to <code>.true.</code> to enable scale variation. It performs a variation of the scales used in eq. (3) by a factor of two so that it surveys the additional possibilities,
	$(2\mu_{\text{ren}}, 2\mu_{\text{fac}}), (\mu_{\text{ren}}/2, \mu_{\text{fac}}/2), \\ (2\mu_{\text{ren}}, \mu_{\text{fac}}), (\mu_{\text{ren}}/2, \mu_{\text{fac}}), (\mu_{\text{ren}}, 2\mu_{\text{fac}}), (\mu_{\text{ren}}, \mu_{\text{fac}}/2). \quad (4)$
	The histograms corresponding to these different choices are included in the output file, from which an envelope of theoretical uncertainty may be constructed by the user.
<code>maxscalevar</code>	Number of additional scale variation points to choose, can be set to two or six. For two it just samples the first two variations as in eq. 4.

Section masses	Description
<code>hmass</code>	Higgs mass
<code>mt</code>	Top-quark mass
<code>mb</code>	Bottom-quark mass
<code>mc</code>	Charm-quark mass
<code>wmass</code>	W-boson mass

Section basicjets	Description
<code>inclusive</code>	This logical parameter chooses whether the calculated cross-section should be inclusive in the number of jets found at NLO. An <i>exclusive</i> cross-section contains the same number of jets at next-to-leading order as at leading order. An <i>inclusive</i> cross-section may instead contain an extra jet at NLO.
<code>algorithm</code>	This specifies the jet-finding algorithm that is used, and can take the values <code>ktal</code> (for the Run II $k_T$ -algorithm), <code>an kt</code> (for the “anti- $k_T$ ” algorithm [25]), <code>cone</code> (for a midpoint cone algorithm), <code>hqrk</code> (for a simplified cone algorithm designed for heavy quark processes) and <code>none</code> (to specify no jet clustering at all). The latter option is only a sensible choice when the leading order cross-section is well-defined without any jet definition: e.g. the single top process, $q\bar{q}' \rightarrow t\bar{b}$ , which is finite as $p_T(\bar{b}) \rightarrow 0$ .
<code>ptjetmin</code> , <code>etajetmax</code> <code>etajetmin</code>	These specify the values of $p_{T,\min}$ and $ \eta _{\max}$ for the jets that are found by the algorithm.
<code>Rcutjet</code>	Optional parameter for setting a minimum jet rapidity $ \eta _{\min}$ . If the final state of the chosen process contains either quarks or gluons then for each event an attempt will be made to form them into jets. For this it is necessary to define the jet separation $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ so that after jet combination, all jet pairs are separated by $\Delta R > R_{\text{cutjet}}$ .
Section masscuts	Description
<code>m34min</code> , <code>m34max</code> , <code>m56min</code> , <code>m56max</code> , <code>m3456min</code> , <code>m3456max</code>	These parameters represent a basic set of mass cuts that are be applied to the calculated cross-section. The only events that contribute to the cross-section will have, for example, $m_{34\min} < m_{34} < m_{34\max}$ where $m_{34}$ is the invariant mass of particles 3 and 4 that are specified by <code>nproc</code> . $m_{34\min} > 0$ is obligatory for processes which can involve a virtual photon, such as <code>nproc=31</code> . By default, the maximum settings are set to $\sqrt{s}$ .

dynamic scale	$\mu_0^2$	comments
m(34)	$(p_3 + p_4)^2$	
m(345)	$(p_3 + p_4 + p_5)^2$	
m(3456)	$(p_3 + p_4 + p_5 + p_6)^2$	
sqrt(M <sup>2</sup> +pt34 <sup>2</sup> )	$M^2 + (p_{T3} + p_{T4})^2$	$M = \text{mass of particle 3+4}$
sqrt(M <sup>2</sup> +pt345 <sup>2</sup> )	$M^2 + (p_{T3} + p_{T4} + p_{T5})^2$	$M = \text{mass of particle 3+4+5}$
sqrt(M <sup>2</sup> +pt5 <sup>2</sup> )	$M^2 + p_{T5}^2$	$M = \text{mass of particle 3+4}$
sqrt(M <sup>2</sup> +ptj1 <sup>2</sup> )	$M^2 + p_{Tj_1}^2$	$M = \text{mass(3+4), } j_1 = \text{leading } p_T \text{ jet}$
pt(photon)	$p_{T\gamma}^2$	
pt(j1)	$p_{Tj_1}^2$	
HT	$\sum_{i=1}^n p_{Ti}$	$n \text{ particles (partons, not jets)}$

Table 11: Choices of the input parameter `dynamicScale` that result in an event-by-event calculation of all relevant scales using the given reference scale-squared  $\mu_0^2$ .

Section cuts	Description
<code>makecuts</code>	If this parameter is set to <code>.false.</code> then no additional cuts are applied to the events and the remaining parameters in this section are ignored. Otherwise, events will be rejected according to a set of cuts that is specified below. Further options may be implemented by editing <code>src/User/gencuts_user.f90</code> .
<code>ptleptmin,</code> <code>etaleptmax</code>	These specify the values of $p_{T,\min}$ and $ \eta _{\max}$ for one of the leptons produced in the process. One can also introduce optional settings <code>ptleptmin</code> and <code>etaleptmin</code> .
<code>etaleptveto</code>	This should be specified as a pair of double precision numbers that indicate a rapidity range that should be excluded for the lepton that passes the above cuts.
<code>ptminmiss</code>	Specifies the minimum missing transverse momentum (coming from neutrinos).
<code>ptlept2min,</code> <code>etalept2max</code>	These specify the values of $p_{T,\min}$ and $ \eta _{\max}$ for the remaining leptons in the process. This allows for staggered cuts where, for instance, only one lepton is required to be hard and central. One can also introduce optional settings <code>ptlept2max</code> and <code>etalept2min</code> .
<code>etalept2veto</code>	This should be specified as a pair of double precision numbers that indicate a rapidity range that should be excluded for the remaining leptons.
<code>m34transmin</code>	For general processes, this specifies the minimum transverse mass of particles 3 and 4,

$$\text{general : } 2p_T(3)p_T(4) \left( 1 - \frac{\vec{p}_T(3) \cdot \vec{p}_T(4)}{p_T(3)p_T(4)} \right) > \text{m34transmin} \quad (5)$$

For the  $W(\rightarrow \ell\nu)\gamma$  process the role of this cut changes, to become instead a cut on the transverse cluster mass of the  $(\ell\gamma, \nu)$  system,

$$W\gamma : \quad \left[ \sqrt{m_{\ell\gamma}^2 + |\vec{p}_T(\ell) + \vec{p}_T(\gamma)|^2} + p_T(\nu) \right]^2 - |\vec{p}_T(\ell) + \vec{p}_T(\gamma) + \vec{p}_T(\nu)|^2 > \text{m34transmin}^2 \quad (6)$$

For the  $Z\gamma$  process this parameter specifies a simple invariant mass cut,

$$Z\gamma : \quad m_{Z\gamma} > \text{m34transmin} \quad (7)$$

A final mode of operation applies to the  $W\gamma$  process and is triggered by a negative value of `m34transmin`. This allows simple access to the cut that was employed in v6.0 of the code:

$$W\gamma, \text{ obsolete : } \quad \begin{aligned} & [p_T(\ell) + p_T(\gamma) + p_T(\nu)]^2 \\ & - |\vec{p}_T(\ell) + \vec{p}_T(\gamma) + \vec{p}_T(\nu)|^2 > |\text{m34transmin}| \end{aligned} \quad (8)$$

In each case the screen output indicates the cut that is applied.

Section cuts	Description
Rjlmin	Using the definition of $\Delta R$ above, requires that all jet-lepton pairs are separated by $\Delta R > R(\text{jet}, \text{lept})_{\text{min}}$ .
Rllmin	When non-zero, all lepton-lepton pairs must be separated by $\Delta R > R(\text{lept}, \text{lept})_{\text{min}}$ .
delyjjmin	This enforces a pseudo-rapidity gap between the two hardest jets $j_1$ and $j_2$ , so that: $ \eta^{j_1} - \eta^{j_2}  > \text{Delta\_eta}(\text{jet}, \text{jet})_{\text{min}}$ .
jetsopphem	If this parameter is set to <code>.true.</code> , then the two hardest jets are required to lie in opposite hemispheres, $\eta^{j_1} \cdot \eta^{j_2} < 0$ .
lbjscheme	This integer parameter provides no additional cuts when it takes the value 0. When equal to 1 or 2, leptons are required to lie between the two hardest jets. With the ordering $\eta^{j^-} < \eta^{j^+}$ for the pseudo-rapidities of jets $j_1$ and $j_2$ : <code>lbjscheme = 1</code> : $\eta^{j^-} < \eta^{\text{leptons}} < \eta^{j^+}$ ; <code>lbjscheme = 2</code> : $\eta^{j^-} + R_{\text{cutjet}} < \eta^{\text{leptons}} < \eta^{j^+} - R_{\text{cutjet}}$ .
ptbjetmin, etabjetmax	If a process involving $b$ -quarks is being calculated, then these can be used to specify <i>stricter</i> values of $p_T^{\text{min}}$ and $ \eta ^{\text{max}}$ for $b$ -jets. Similarly, values for <code>ptbjetmax</code> and <code>etabjetmin</code> can be specified.

Section photon	Description
fragmentation	This parameter is a logical variable that determines whether the production of photons by a parton fragmentation process is included. If <code>fragmentation</code> is set to <code>.true.</code> the code uses a standard cone isolation procedure (that includes LO fragmentation contributions in the NLO calculation). If <code>fragmentation</code> is set to <code>.false.</code> the code implements a Frixione-style photon cut [24], $\sum_{i \in R_0} E_{T,i}^j < \epsilon_h E_T^\gamma \left( \frac{1 - \cos R_{i\gamma}}{1 - \cos R_0} \right)^n . \quad (9)$ In this equation, $R_0$ , $\epsilon_h$ and $n$ are defined by <code>cone_ang</code> , <code>epsilon_h</code> and <code>n_pow</code> respectively (see below). $E_{T,i}^j$ is the transverse energy of a parton, $E_T^\gamma$ is the transverse energy of the photon and $R_{i\gamma}$ is the separation between the photon and the parton using the usual definition $R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ . $n$ is an integer parameter which by default is set to 1.
fragmentation_set	A length eight character variable that is used to choose the particular photon fragmentation set. Currently implemented fragmentation functions can be called with ‘BFGSet_I’, ‘BFGSetII’ [26] or ‘GdRG_LO’ [27].
fragmentation_scale	A double precision variable that will be used to choose the scale at which the photon fragmentation is evaluated.
gamptmin	This specifies the value of $p_T^{\text{min}}$ for the photon with the largest transverse momentum. Note that this cut, together with all the photon cuts specified in this section of the input file, are applied even if <code>makecuts</code> is set to <code>.false.</code> . One can also add an entry for <code>gamptmax</code> to cut on a range.

<code>gammrapmax</code>		This specifies the value of $ y ^{\max}$ for any photons produced in the process. One can also add an entry for <code>gammrapmin</code> to cut on a range.
<code>gammpt2</code>	and	These specify the values of $p_T^{\min}$ for the second and third photons, ordered by $p_T$ .
<code>gammpt3</code>		
<code>Rgalmin</code>		Using the usual definition of $\Delta R$ , this requires that all photon-lepton pairs are separated by $\Delta R > \text{Rgalmin}$ . This parameter must be non-zero for processes in which photon radiation from leptons is included.
<code>Rgagamin</code>		Using the usual definition of $\Delta R$ , this requires that all photon pairs are separated by $\Delta R > \text{Rgagamin}$ .
<code>Rgajetmin</code>		Using the usual definition of $\Delta R$ , this requires that all photon-jet pairs are separated by $\Delta R > \text{Rgajetmin}$ .
<code>cone_ang</code>		Fixes the cone size ( $R_0$ ) for photon isolation. This cone is used in both forms of isolation.
<code>epsilon_h</code>		This cut controls the amount of radiation allowed in cone when <code>fragmentation</code> is set to <code>.true.</code> . If <code>epsilon_h &lt; 1</code> then the photon is isolated using $\sum_{\in R_0} E_T(\text{had}) < \epsilon_h p_T^\gamma$ . Otherwise <code>epsilon_h &gt; 1</code> sets $E_T(\text{max})$ in $\sum_{\in R_0} E_T(\text{had}) < E_T(\text{max})$ . If the user wishes to always use a scaling or fixed isolation cut, independent of the value of <code>epsilon_h</code> , the routine <code>src/Cuts/iso.f</code> may be edited and the value of the variable <code>imode</code> changed according to the comments. When <code>fragmentation</code> is set to <code>.false.</code> , $\epsilon_h$ controls the amount of hadronic energy allowed inside the cone using the Frixione isolation prescription (see above, Eq. (9)).
<code>n_pow</code>		When using the Frixione isolation prescription, the exponent $n$ in Eq. (9).

### Section histogram

### Description

<code>writetop</code>	Write output histograms suitable as input for top-drawer.
<code>writetxt</code>	Write output histograms as whitespace-separated columns.

### Section integration

### Description

<code>usesobol</code>	When <code>.true.</code> and the number of MPI processes is a power of two, the Sobol sequence is used, see ref. [1], otherwise the MT19937 pseudo random number generator.
<code>seed</code>	Initialization seed for MT19937 pseudo random number generator.
<code>precisiongoal</code>	Relative precision goal for the integration.
<code>readin</code>	When <code>.true.</code> the automatically written snapshot from a previous run will be read-in to resume the integration.
<code>writeintermediate</code>	When <code>.true.</code> histograms are written after each Vegas iteration.
<code>warmupprecisiongoal</code>	Sets the relative precision goal for the warmup run. Unless this precision is reached, the number of calls for the warmup is increased.
<code>warmupchisqgoal</code>	Sets the $\chi^2$ per iteration goal for the warmup run. Unless the $\chi^2/\text{it.}$ of the warmup is below this target, the number of calls for the warmup is increased.

## 4.6. Process specific options

Section	Description
singletop	
<code>c_phiq</code>	Sets real Wilson coefficient of $Q_{\varphi q}^{(3,33)}$ for processes 164 and 169. See section 6.37 and ref. [28].
<code>c_phiphi</code>	Sets real and imaginary part of the $Q_{\varphi ud}^{33}$ Wilson coefficient.
<code>c_tw</code>	Sets real and imaginary part of the $Q_{uW}^{33}$ Wilson coefficient.
<code>c_bw</code>	Sets real and imaginary part of the $Q_{dW}^{33}$ Wilson coefficient.
<code>c_tg</code>	Sets real and imaginary part of the $Q_{uG}^{33}$ Wilson coefficient.
<code>c_bg</code>	Sets real and imaginary part of the $Q_{dG}^{33}$ Wilson coefficient.
<code>lambda</code>	Scale $\Lambda$ , see section 6.37 and ref. [28].
<code>enable_lambda4</code>	Enable contributions of order $1/\Lambda^4$ when set to <code>.true.</code>
<code>disable_sm</code>	When set to <code>.true.</code> the pure SM contributions are disabled, and just the SM-EFT interference and EFT contributions are calculated.
<code>mode_anomcoup</code>	When set to <code>.true.</code> at LO one can reproduce results obtained without power counting as in the anomalous couplings approach, see section 6.37 and ref. [28].
anom_wz	
Section	Description
<code>enable</code>	Boolean flag to enable anomalous W-boson and Z-boson coupling contributions for certain processes. False has the same effect as setting all anomalous couplings to zero, but additionally skips computation of anomalous coupling code parts.
<code>delg1_z</code>	$\Delta g_1^Z$ See section 6.22.
<code>delk_z</code>	$\Delta \kappa^Z$ See section 6.22.
<code>delk_g</code>	$\Delta \kappa^\gamma$ See sections 6.22 and 6.61.
<code>lambda_z</code>	$\Lambda^Z$ See section 6.22.
<code>lambda_g</code>	$\Lambda^\gamma$ See sections 6.22 and 6.61.
<code>h1Z</code>	$h_1^Z$ Anomalous couplings for $Z\gamma$ process at NNLO. See section 6.62.
<code>h1gam</code>	$h_1^\gamma$ See section 6.62.
<code>h2Z</code>	$h_2^Z$ See section 6.62.
<code>h2gam</code>	$h_2^\gamma$ See section 6.62.
<code>h3Z</code>	$h_3^Z$ See section 6.62.
<code>h3gam</code>	$h_3^\gamma$ See section 6.62.
<code>h4Z</code>	$h_4^Z$ See section 6.62.
<code>h4gam</code>	$h_4^\gamma$ See section 6.62.
<code>tevscale</code>	Form-factor scale, in TeV See section 6.22. No form-factors are applied to the anomalous couplings if this value is negative.



Section <code>wz2jet</code>	Description
<code>Qflag</code>	This only has an effect when running a $W + 2$ jets or $Z + 2$ jets process. Please see section 6.8 below.
<code>Gflag</code>	This only has an effect when running a $W + 2$ jets or $Z + 2$ jets process. Please see section 6.8 below.

Section <code>hjetmass</code>	Description
<code>mtex</code>	Sets the order $k = 0, 2, 4$ of the $1/m_t^k$ expansion for virtual corrections in the $H$ +jet process 200. See section 6.39.

Section <code>anom_higgs</code>	Description
<code>hwidth_ratio</code>	For processes 123–126, 128–133 only, this variable provides a rescaling of the width of the Higgs boson. Couplings are rescaled such that the corresponding cross section close to the Higgs boson peak is unchanged. Further details of this procedure are given in <a href="#">arXiv:1311.3589</a> .
<code>cttH</code> , <code>cWWH</code>	See <a href="#">arXiv:1311.3589</a> .

Section <code>extra</code>	Description
<code>debug</code>	A logical variable which can be used during a debugging phase to mandate special behaviours. Passed by common block <code>common/debug/debug</code> .
<code>verbose</code>	A logical variable which can be used during a debugging phase to write special information. Passed in common block <code>common/verbose/verbose</code> .
<code>new_pspace</code>	A logical variable which can be used during a debugging phase to test alternative versions of the phase space. Passed in common block <code>common/new_pspace/new_pspace</code> .
<code>spira</code>	A variable. If <code>spira</code> is true, we calculate the width of the Higgs boson by interpolating from a table calculated using the NLO code of M. Spira. Otherwise the LO value valid for low Higgs masses only is used.
<code>noglu</code>	A logical variable. The default value is false. If set to true, no processes involving initial gluons are included.
<code>ggonly</code>	A logical variable. The default value is false. If set to true, only the processes involving initial gluons in both hadrons are included.
<code>gqonly</code>	The default value is false. If set to true, only the processes involving an initial gluon in one hadron and an initial quark or antiquark in the other hadron (or vice versa) are included.
<code>omitgg</code>	A logical variable. The default value is false. If set to true, the gluon-gluon initial state is not included.
<code>clustering</code>	This logical parameter determines whether clustering is performed to yield jets. Only during a debugging phase should this variable be set to false.

<code>colourchoice</code>	If <code>colourchoice=0</code> , all colour structure are included ( $W, Z + 2$ jets). If <code>colourchoice=1</code> , only the leading colour structure is included ( $W, Z + 2$ jets).
<code>rtsmin</code>	A minimum value of $\sqrt{s_{12}}$ , which ensures that the invariant mass of the incoming partons can never be less than <code>rtsmin</code> .
<code>cutoff</code>	Cutoff according to <code>src/Need/smallnew.f</code> and <code>src/Need/smalltau.f</code>

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Section dipoles	Description
<code>aii</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of initial-initial dipoles as suggested by Trocsanyi and Nagy [29]. The value <code>aii=1</code> corresponds to standard Catani-Seymour subtraction.
<code>aif</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of initial-final dipoles as suggested by Trocsanyi and Nagy [29]. The value <code>aif=1</code> corresponds to standard Catani-Seymour subtraction.
<code>afi</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of final-initial dipoles as suggested by Trocsanyi and Nagy [29]. The value <code>afi=1</code> corresponds to standard Catani-Seymour subtraction.
<code>aff</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of final-final dipoles as suggested by Trocsanyi and Nagy [29]. The value <code>aff=1</code> corresponds to standard Catani-Seymour subtraction.
<code>bfi</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of final-initial dipoles in the photon fragmentation case.
<code>bff</code>	A double precision variable which can be used to limit the kinematic range for the subtraction of final-final dipoles in the photon fragmentation case.

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#### 4.7. User modifications to the code

Modifying the plotting routines in the files `src/User/nplotter*.f` allows for modification of the pre-defined histograms and addition of any number of arbitrary observables. The routine `gencuts_user` can be adjusted in the file `src/User/gencuts_user.f90` for additional cuts after the jet algorithm has performed the clustering. In the same file the routine `reweight_user` can be modified to include a manual re-weighting for all integral contributions. This can be used to obtain improved uncertainties in, for example, tails of distributions. One example is included in the subdirectory `examples`, where the `reweight_user` function approximately flattens the Higgs transverse momentum distribution, leading to equal relative uncertainties even in the tail at 1 TeV.

- subroutine `nplotter_user(pjet, wt, wt2, nd)` This subroutine is called to allow the user to bin their own histograms. Variables passed to this routine:

- p: 4-momenta of incoming partons(i=1,2), outgoing leptons and jets in the format p(i,4) with the particles numbered according to the input file and components labelled by (px,py,pz,E).
- wt: weight of this event
- wt2: weight<sup>2</sup> of this event
- nd: an integer specifying the dipole number of this contribution (if applicable), otherwise equal to zero.

## 5. Benchmark results at NNLO

We perform benchmark calculations with the default set of EW parameters and for the LHC operating at  $\sqrt{s} = 14$  TeV. We allow all vector bosons to be off-shell (`zerowidth` is `.false.`) and include their decays (`removebr` is `.false.`). For each Higgs boson process we consider the decay  $H \rightarrow \tau^- \tau^+$ . For parameters that are set in the input file we use,

$$m_H = 125 \text{ GeV}, \quad m_t = 173.3 \text{ GeV}, \quad m_b = 4.66 \text{ GeV}, \quad (10)$$

and we use the NNLO CT14 pdf set (i.e. `pdlabel` is `CT14.NN`) with  $\mu_F = \mu_R = Q^2$  (i.e. we set `dynamicsscale` equal to either `m(34)` or `m(345)` or `m(3456)`, as appropriate). Our generic set of cuts is,

$$\begin{aligned} p_T(\text{lepton}) &> 20 \text{ GeV}, & |\eta(\text{lepton})| &< 2.4, \\ p_T(\text{photon } 1) &> 40 \text{ GeV}, & p_T(\text{photon } 2) &> 25 \text{ GeV}, \\ |\eta(\text{photon})| &< 2.5, & \Delta R(\text{photon } 1, \text{photon } 2) &> 0.4, \\ E_T^{\text{miss}} &> 30 \text{ GeV}, & \Delta R(\text{photon}, \text{lepton}) &> 0.3 \end{aligned} \quad (11)$$

For  $Z$  production we also impose a minimum  $Z^*$  virtuality (`m34min`) of 40 GeV.

Our benchmark results are shown in Table 28 and were performed on an Intel Xeon X5650 @ 2.67GHz system with 16 nodes of 12 cores each. MCFM was compiled with the Intel compiler and default optimizations as well as the default MCFM setup including all pre-defined histograms. The NNLO CPU time includes the time necessary for the NLO calculation. The numbers for the NLO and NNLO results were obtained independently. By tweaking the initial number of calls or the number of iterations per batch it is certainly possible to optimize the runtimes. While the numerical precision is not yet good sufficient for most of the fitted corrections to significantly improve the results, the fits are highly reliable and correctly estimate the residual  $\tau_{\text{cut}}$  dependence.

## 6. Notes on specific processes

The processes described in the file `process.DAT` include appropriate boson decays when the parameter `removebr` is set to `.false.`. In many cases a more simple calculation can be performed by setting this parameter to `.true.`, in which case these decays are not performed. Technically the full calculation including the decays is still performed but cuts are not

Table 27: Benchmark cross-sections at NLO and NNLO, using the parameters and settings described in the text. All numbers are obtained for a numerical 0.2% precision goal. All NLO numbers are obtained within minutes on a desktop system, except for  $Z\gamma$ , which requires at the order of 20-30 minutes. The NNLO CPU time includes the time for the NLO calculation.

Process	nproc	$\tau_{\text{cut}}$ [GeV]	$\sigma^{\text{NLO}}$	$\sigma^{\text{NNLO}}$	fitted corr.	CPU time [h]
$W^+$	1	$6 \cdot 10^{-3} m_W$	4.221 nb	$4.209 \pm 0.005$ nb	$-27 \pm 15$ pb	7.6
$W^-$	6	$6 \cdot 10^{-3} m_W$	3.315 nb	$3.275 \pm 0.004$ nb	$-25 \pm 10$ pb	7.8
$Z$	31	$6 \cdot 10^{-3} m_Z$	885.3 pb	$875.8 \pm 0.9$ nb	$-3.5 \pm 2.0$ fb	13.0
$H$	112	$4 \cdot 10^{-3} m_H$	1.396 pb	$1.872 \pm 0.002$ pb	$7 \pm 6$ fb	9.7
$\gamma\gamma$	285	$1 \cdot 10^{-4} m_{\gamma\gamma}$	27.91 pb	$43.54 \pm 0.08$ pb	$0.36 \pm 0.10$ pb	83.2
$W^+H$	91	$3 \cdot 10^{-3} m_{W^+H}$	2.204 fb	$2.262 \pm 0.004$ fb	$0.002 \pm 0.008$ fb	16.0
$W^-H$	96	$3 \cdot 10^{-3} m_{W^-H}$	1.491 fb	$1.526 \pm 0.003$ fb	$-0.005 \pm 0.007$ fb	13.0
$ZH$	110	$3 \cdot 10^{-3} m_{ZH}$	0.753 fb	$0.842 \pm 0.001$ fb	$-0.005 \pm 0.003$ fb	12.5
$Z\gamma$	300	$3 \cdot 10^{-4} m_{Z\gamma}$	434 fb	$525.5 \pm 1.0$ fb	$4.5 \pm 1.7$ fb	202.5

Table 28: Benchmark cross-sections at NLO and NNLO, using the parameters and settings described in the text.  $\delta\sigma^{MC}$  represents the uncertainty from the integration, while  $\delta\sigma^{pc}$  is an estimate of the uncertainty due to neglected power corrections at NNLO.

Process	nproc	$\sigma_{\text{NLO}} \pm \delta\sigma_{\text{NLO}}^{\text{MC}}$	$\sigma_{\text{NNLO}} \pm \delta\sigma_{\text{NNLO}}^{\text{MC}} \pm \delta\sigma_{\text{NNLO}}^{\text{pc}}$
$W^+$	1	$4.220 \pm 0.002$ nb	$4.19 \pm 0.02 \pm 0.043$ nb
$W^-$	6	$3.315 \pm 0.001$ nb	$3.23 \pm 0.01 \pm 0.033$ nb
$Z$	31	$885.2 \pm 0.3$ pb	$878 \pm 3 \pm 9$ pb
$H$	112	$1.395 \pm 0.001$ pb	$1.865 \pm 0.004 \pm 0.019$ pb
$\gamma\gamma$	285	$27.94 \pm 0.01$ pb	$43.60 \pm 0.06 \pm 0.44$ pb
$W^+H$	91	$2.208 \pm 0.002$ fb	$2.268 \pm 0.007 \pm 0.023$ fb
$W^-H$	96	$1.494 \pm 0.001$ fb	$1.519 \pm 0.004 \pm 0.015$ fb
$ZH$	110	$0.7535 \pm 0.0004$ fb	$0.846 \pm 0.001 \pm 0.0085$ fb
$Z\gamma$	300	$959 \pm 8$ fb	$1268 \pm 22$ fb

performed on the decay products and the branching ratio is divided out, thus yielding the cross section before decay. In the notes below we indicate the simpler processes thus obtained. When running in this mode, the parameter `zerowidth` should be set to `.true.` for consistency. However in certain circumstances, for the sake of comparison, it may be useful to run with it set to `.false..`

### 6.1. $W$ -boson production, processes 1,6

These processes represent the production of a  $W$  boson which subsequently decays leptonically. The calculation may be performed at NLO.

When `removebr` is true, the  $W$  boson does not decay.

### 6.2. $W + \text{jet}$ production, processes 11,16

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a single jet. The calculation may be performed at NLO.

When `removebr` is true, the  $W$  boson does not decay.

### 6.3. $W + b$ production, processes 12,17

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a single bottom quark, exploiting the weak transitions  $c \rightarrow b$  and  $u \rightarrow b$ . This is produced at leading order by an initial state which contains a charm quark (or the CKM suppressed  $u$  quark) and a gluon. The effect of the bottom quark mass is included throughout the calculation. For this case the CKM matrix elements  $V_{cb}$  and  $V_{ub}$ , (if they are equal to zero in the input data file, `mdata.f`) are set equal to 0.041 and 0.00347 respectively. Otherwise the non-zero values specified in `mdata.f` are used. The calculation of this process may be performed at NLO.

When `removebr` is true, the  $W$  boson does not decay.

### 6.4. $W + c$ production, processes 13,18

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a charm quark. This is produced at leading order by an initial state which contains a strange quark (or Cabibbo suppressed  $d$  quark) and a gluon. The effect of the charm quark mass is included throughout the calculation. As of version 5.2, the calculation of this process may be performed at NLO.

When `removebr` is true, the  $W$  boson does not decay.

### 6.5. $W + c$ production ( $m_c = 0$ ), processes 14,19

These processes are identical to 13 and 18 except for the fact that the charm quark mass is neglected. The calculation can currently be performed at LO only.

## 6.6. $W + b\bar{b}$ production, processes 20,25

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a  $b\bar{b}$  pair. The effect of the bottom quark mass is included throughout the calculation. Beginning with MCFM version 6.0 this calculation may be performed at NLO, thanks to the incorporation of the virtual corrections from ref. [30]. When `removebr` is true, the  $W$  boson does not decay.

To select final states in which one of the  $b$ -quarks may be unobserved the user can employ processes 401–408 instead (see section 6.71). These processes use the same matrix elements but make specific requirements on the kinematics of the  $b$ -quarks and QCD radiation.

## 6.7. $W + b\bar{b}$ production ( $m_b = 0$ ), processes 21,26

These processes are identical to 20 and 25 except for the fact that the bottom quark mass is neglected. This allows the calculation to be performed up to NLO, with currently calculated virtual matrix elements. These processes run considerably faster than the corresponding processes with the mass for the  $b$  quark, (20,25). In circumstances where both  $b$  quarks are at large transverse momentum, the inclusion of the mass for the  $b$ -quark is not mandatory and a good estimate of the cross section may be obtained by using these processes.

When `removebr` is true, the  $W$  boson does not decay.

## 6.8. $W + 2$ jets production, processes 22,27

*[For more details on this calculation, please refer to  
hep-ph/0202176 and hep-ph/0308195]*

This process represents the production of a  $W$  boson and 2 jets, where the  $W$  boson decays leptonically. The calculation may be performed up to NLO, as detailed below. Virtual amplitudes are taken from ref. [31].

For these processes (and also for  $Z + 2$  jet production, `nproc=44,46`) the next-to-leading order matrix elements are particularly complex and so they have been divided into two groups. The division is according to the lowest order diagrams from which they originate:

1. Diagrams involving two external quark lines and two external gluons, the “`Gflag`” contribution. The real diagrams in this case thus involve three external gluons.
2. Diagrams where all four external lines are quarks, the “`Qflag`” contribution. The real diagrams in this case involve only one gluon.

By specifying `Gflag` and `Qflag` in the file `input.ini` one may select one of these options at a time. The full result may be obtained by straightforward addition of the two individual pieces, with no meaning attached to either piece separately. Both of these may be set to `.true.` simultaneously, however this may result in lengthy run-times for sufficient convergence of the integral.

When `removebr` is true, the  $W$  boson does not decay.

### 6.9. $W + 3$ jets production, processes 23,28

This process represents the production of a  $W$  boson and 3 jets, where the  $W$  boson decays leptonically. The calculation may be performed at LO only.

When `removebr` is true, the  $W$  boson does not decay.

### 6.10. $W + b\bar{b} +$ jet production ( $m_b = 0$ ), processes 24,29

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a  $b\bar{b}$  pair and an additional jet. The effect of the bottom quark mass is neglected throughout and the calculation may be performed at LO only.

When `removebr` is true, the  $W$  boson does not decay.

### 6.11. $Z$ -boson production, processes 31–33

These processes represent the production of a  $Z$  boson which subsequently decays either into electrons (`nproc=31`), neutrinos (`nproc=32`) or bottom quarks (`nproc=33`). Where appropriate, the effect of a virtual photon is also included. As noted above, in these latter cases `m34min > 0` is obligatory. The calculation may be performed at NLO, although the NLO calculation of process 33 does not include radiation from the bottom quarks (i.e. radiation occurs in the initial state only).

When `removebr` is true in process 31, the  $Z$  boson does not decay.

### 6.12. $Z$ -boson production decaying to jets, processes 34–35

Radiation from the final state quarks is not included in this process.

### 6.13. $t\bar{t}$ production mediated by $Z/\gamma^*$ -boson exchange, process 36

These processes represent the production of a virtual  $Z$  boson or photon which subsequently decays into  $t\bar{t}$ . The leptonic decays of the top quarks are included. Switching `zerowidth` from `.true.` to `.false.` only affects the  $W$  bosons from the top quark decay. Note that `m34min > 0` is obligatory due to the inclusion of the virtual photon diagrams. The calculation may be only be performed at LO.

### 6.14. Lepton pair production through photonic initial states, process 310

This process represents the production of a lepton pair through an electroweak process involving two photons in the initial state,  $\gamma\gamma \rightarrow e^-e^+$ .

### 6.15. $Z + \text{jet}$ production, processes 41–43

These processes represent the production of a  $Z$  boson and a single jet, where the  $Z$  subsequently decays either into electrons (`nproc=41`), neutrinos (`nproc=42`) or bottom quarks (`nproc=43`). Where appropriate, the effect of a virtual photon is also included. The calculation may be performed at NLO, although the NLO calculation of process 43 does not include radiation from the bottom quarks.

When `removebr` is true in process 41, the  $Z$  boson does not decay.

### 6.16. $Z + 2 \text{jets}$ production, processes 44, 46

*[For more details on this calculation, please refer to  
hep-ph/0202176 and hep-ph/0308195]*

These processes represents the production of a  $Z$  boson and 2 jets, including also the effect of a virtual photon (`nproc=44` only). The  $Z/\gamma^*$  decays to an  $e^+e^-$  pair (`nproc=44`) or into three species of neutrino (`nproc=46`). The calculation may be performed up to NLO – please see the earlier Section 6.8 for more details, especially the discussion regarding `Qflag` and `Gflag`. As of version 6.0, both of these may be set to `.true.` simultaneously but this may result in lengthy run-times for sufficient convergence of the integral. Virtual amplitudes are taken from ref. [31].

When `removebr` is true, the  $Z$  boson does not decay.

### 6.17. $Z + 3 \text{jets}$ production, processes 45, 47

These processes represent the production of a  $Z$  boson and 3 jets, including also the effect of a virtual photon (`nproc=45` only). The  $Z/\gamma^*$  decays to an  $e^+e^-$  pair (`nproc=45`) or into three species of neutrino (`nproc=47`). The calculation may be performed at LO only.

When `removebr` is true, the  $Z$  boson does not decay.

### 6.18. $Z + b\bar{b}$ production, process 50

These processes represent the production of a  $Z$  boson (or virtual photon) which subsequently decays leptonically, in association with a  $b\bar{b}$  pair. The effect of the bottom quark mass is included throughout the calculation. The calculation may be performed at LO only.

When `removebr` is true, the  $Z$  boson does not decay.



### 6.19. $Z + b\bar{b}$ production ( $m_b = 0$ ), processes 51–53

Process 51 is identical to 50 except for the fact that the bottom quark mass is neglected. This allows the calculation to be performed up to NLO. The other processes account for the decays into neutrinos (`nproc=52`) and bottom quarks (`nproc=53`). Note that the NLO calculation of process 53 does not currently include radiation from the bottom quarks produced in the decay.

When `removebr` is true in process 51, the  $Z$  boson does not decay.

### 6.20. $Z + b\bar{b} + \text{jet}$ production ( $m_b = 0$ ), process 54

This process represents the production of a  $Z$  boson (and virtual photon) which subsequently decays leptonically, in association with a  $b\bar{b}$  pair and an additional jet. The effect of the bottom quark mass is neglected throughout and the calculation may be performed at LO only.

When `removebr` is true, the  $Z$  boson does not decay.

### 6.21. $Z + c\bar{c}$ production ( $m_c = 0$ ), process 56

Process 56 is the equivalent of 51, with the bottom quarks replaced by charm. Although the charm mass is neglected, the calculation contains diagrams with two gluons in the initial state and a  $Z$  coupling to the heavy quark line – hence the dependence upon the quark flavour.

When `removebr` is true in process 56, the  $Z$  boson does not decay.

### 6.22. Di-boson production, processes 61–89

*[For more details on these calculations, please refer to hep-ph/9905386 and arXiv:1105.0020 [hep-ph]]*

These processes represent the production of a diboson pair  $V_1V_2$ , where  $V_1$  and  $V_2$  may be either a  $W$  or  $Z/\gamma^*$ . All the processes in this section may be calculated at NLO, with the exception of `nproc=66,69`. There are various possibilities for the subsequent decay of the bosons, as specified in the sections below. Amplitudes are taken from ref. [32]. Where appropriate, these processes include glue-gluon initiated box diagrams which first contribute at order  $\alpha_s^2$  but are included here in the NLO calculation. We also include singly resonant diagrams at NLO for all processes in the case `zerowidth = .false..`

For processes 62, 63, 64, 65, 74 and 75 the default behaviour is that the hadronic decay products of the bosons are clustered into jets using the supplied jet algorithm parameters, but no cut is applied on the number of jets. This behaviour can be altered by changing the value of the variable `notag` in the file `src/User/setnotag.f`.

### 6.22.1. $WW$ production, processes 61–64, 69

For  $WW$  production, both  $W$ 's can decay leptonically (`nproc=61`) or one may decay hadronically (`nproc=62` for  $W^-$  and `nproc=64` for  $W^+$ ). Corresponding to processes 62, 64, processes 63, 65 implement radiation in decay from the hadronically decaying  $W$ 's. Process 69 implements the matrix elements for the leptonic decay of both  $W$ 's but where no polarization information is retained. It is included for the sake of comparison with other calculations. Processes 62 and 64 may be run at NLO with the option `todk`, including radiation in the decay of the hadronically decaying  $W$ . Processes 63 and 65 give the effect of radiation in the decay alone by making the choices `virt`, `real` or `tota`.

Note that, in processes 62 and 64, the NLO corrections include radiation from the hadronic decays of the  $W$ .

The NLO calculations include contributions from the process  $gg \rightarrow WW$  that proceeds through quark loops. The calculation of loops containing the third quark generation includes the effect of the top quark mass (but  $m_b = 0$ ), while the first two generations are considered massless. For numerical stability, a small cut on the transverse momentum of the  $W$  bosons is applied:  $p_T(W) > 0.05$  GeV for loops containing massless (first or second generation) quarks,  $p_T(W) > 2$  GeV for  $(t, b)$  loops. This typically removes less than 0.1% of the total cross section. The values of these cutoffs can be changed by editing `src/WW/gg_WW.f` and recompiling.

When `removebr` is true in processes 61 and 69, the  $W$  bosons do not decay.

### 6.22.2. $WW$ +jet production, process 66

This process is only implemented for the leptonic decay modes of both  $W$  bosons and is currently limited to LO accuracy only. When `removebr` is true, the  $W$  bosons do not decay.

### 6.22.3. $WZ$ production, processes 71–80

For  $WZ$  production, the  $W$  is chosen to decay leptonically. The  $Z$  (or virtual photon, when appropriate) may decay into electrons (`nproc=71,76`), neutrinos (`nproc=72,77`), a pair of bottom quarks (`nproc=73,78`), three generations of down-type quarks (`nproc=74,79`) or two generations of up-type quarks (`nproc=75,80`). In process 78 the mass of the  $b$ -quark is neglected. These processes will be observed in the final state as  $W$ -boson + two or three jets. In processes 72 and 77, a sum is performed over all three species of neutrinos.

When `removebr` is true in processes 71 and 76, neither the  $W$  or the  $Z$  boson decays.

### 6.22.4. $ZZ$ production, processes 81–84, 86–90

For  $ZZ$  production, there are two sets of processes corresponding to the inclusion of a virtual photon when appropriate (`nproc=81–84`) and the case where it is neglected (`nproc=86–89`). Thus `nproc=86–89` are really for diagnostic purposes only.

The  $Z$ 's can either both decay leptonically (`nproc=81,86`), one can decay leptonically while the other decays into neutrinos (`nproc=82,87`) or bottom quarks (`nproc=83,88`), or one decays into neutrinos and the other into a bottom quark pair (`nproc=84,89`). In process 83 the mass of the  $b$ -quark is neglected. Note that, in processes 83–84 and 88–89, the NLO corrections do not include radiation from the bottom quarks that are produced by the  $Z$  decay.

The NLO calculations include contributions from the process  $gg \rightarrow ZZ$  that proceeds through quark loops. The calculation of loops containing the third quark generation includes the effect of both the top and the bottom quark mass ( $m_t, m_b \neq 0$ ), while the first two generations are considered massless. For numerical stability, a small cut on the transverse momentum of the  $Z$  bosons is applied:  $p_T(Z) > 0.1$  GeV. This typically removes less than 0.1% of the total cross section. The values of these cutoffs can be changed by editing `src/ZZ/getggZZamps.f` and recompiling.

When `removebr` is true in processes 81 and 86, neither of the  $Z$  bosons decays.

In process 90 the two  $Z$  bosons decay to identical charged leptons, and interference effects between the decay products of the two  $Z$  bosons are included. This process may be calculated at LO only.

#### 6.22.5. $ZZ$ +jet production, process 85

This process is only implemented for the case when one  $Z$  boson decays to electrons and the other to neutrinos (i.e. the companion of `nproc=82`). It may only be calculated at LO. When `removebr` is true, the  $Z$  bosons do not decay.

#### 6.22.6. Anomalous couplings

As of version 3.0, it is possible to specify anomalous trilinear couplings for the  $W^+W^-Z$  and  $W^+W^-\gamma$  vertices that are relevant for  $WW$  and  $WZ$  production. To run in this mode, one must set `zerowidth` equal to `.true.` and modify the appropriate lines for the couplings in `input.ini` (see below). Note that, at present, the effect of anomalous couplings is not included in the gluon-gluon initiated contributions to the  $WW$  process.

The anomalous couplings appear in the Lagrangian,  $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{anom}$  as follows (where  $\mathcal{L}_{SM}$  represents the usual Standard Model Lagrangian and  $\mathcal{L}_{anom}$  is taken from Ref. [33]):

$$\begin{aligned} \mathcal{L}_{anom} = & ig_{WWZ} \left[ \Delta g_1^Z (W_{\mu\nu}^* W^\mu Z^\nu - W_{\mu\nu} W^{*\mu} Z^\nu) + \Delta \kappa^Z W_\mu^* W_\nu Z^{\mu\nu} \right. \\ & \left. + \frac{\lambda^Z}{M_W^2} W_{\rho\mu}^* W_\nu^\mu Z^{\nu\rho} \right] + ig_{WW\gamma} \left[ \Delta \kappa^\gamma W_\mu^* W_\nu \gamma^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_{\rho\mu}^* W_\nu^\mu \gamma^{\nu\rho} \right], \end{aligned}$$

where  $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$  and the overall coupling factors are  $g_{WW\gamma} = -e$ ,  $g_{WWZ} = -e \cot \theta_w$ . This is the most general Lagrangian that conserves  $C$  and  $P$  separately and electromagnetic gauge invariance requires that there is no equivalent of the  $\Delta g_1^Z$  term for the photon coupling.

In order to avoid a violation of unitarity, these couplings are often included only after suppression by dipole form factors,

$$\Delta g_1^Z \rightarrow \frac{\Delta g_1^Z}{(1 + \hat{s}/\Lambda^2)^2}, \quad \Delta \kappa^{Z/\gamma} \rightarrow \frac{\Delta \kappa_1^{Z/\gamma}}{(1 + \hat{s}/\Lambda^2)^2}, \quad \lambda^{Z/\gamma} \rightarrow \frac{\Delta \lambda^{Z/\gamma}}{(1 + \hat{s}/\Lambda^2)^2},$$

where  $\hat{s}$  is the vector boson pair invariant mass and  $\Lambda$  is an additional parameter giving the scale of new physics, which should be in the TeV range. These form factors should be produced by the new physics associated with the anomalous couplings and this choice is somewhat arbitrary. The use of the form factors can be disabled as described below.

The file `input.ini` contains the values of the 6 parameters which specify the anomalous couplings. If the input file contains a negative value for the form-factor scale then the suppression factors described above are not applied.

### 6.23. $WH$ production, processes 91-94, 96-99

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a Standard Model Higgs boson that decays into a bottom quark pair (`nproc=91, 96`), a pair of  $W$  bosons (`nproc=92, 97`), a pair of  $Z$  bosons (`nproc=93, 98`), or a pair of photons (`nproc=94, 99`). Note that in the cases of Higgs decay to  $W, (Z)$  pairs, below the  $W, (Z)$  pair threshold one of the  $W, (Z)$  bosons is virtual and therefore one must set `zerowidth=.false..` The calculation may be performed at NLO. Note that the bottom quarks are considered massless and radiation from the bottom quarks in the decay is not included.

When `removebr` is true, neither the  $W$  boson nor the Higgs decays.

### 6.24. $ZH$ production, processes 101–109

These processes represent the production of a  $Z$  boson (or virtual photon) in association with a Standard Model Higgs boson that decays into a bottom quark pair (`nproc=101-103`), or decays into a pair of photons (`nproc=104-105`) or a pair of  $W$  bosons (`nproc=106-108`), or a pair of  $Z$  bosons (`nproc=109`). The  $Z$  subsequently decays into either an  $e^+e^-$  pair (`nproc=101, 106, 109`), neutrinos (`nproc=102, 107`) or a bottom quark pair (`nproc=103, 108`). The calculation may be performed at NLO, although radiation from the bottom quarks in the decay of the Higgs (or the  $Z$ , for processes 103, 108) is not included.

When `removebr` is true in processes 101, 106, 109, neither the  $Z$  boson nor the Higgs decays.

### 6.25. Higgs production, processes 111–121

These processes represent the production of a Standard Model Higgs boson that decays either into a bottom quark pair (`nproc=111`), a pair of tau's (`nproc=112`), a  $W^+W^-$  pair that further decays leptonically (`nproc=113`) a  $W^+W^-$  pair where the  $W^-$  decays hadronically (`nproc=114, 115`) or a  $ZZ$  pair (`nproc=116-118`). In addition, the loop-level decays of the Higgs into a pair of photons (`nproc=119`) and the  $Z\gamma$  decay are included (`nproc=120, 121`).

For the case of  $W^+W^-$  process `nproc=115` gives the contribution of radiation from the hadronically decaying  $W^-$ . Process 114 may be run at NLO with the option `todk`, including radiation in the decay of the hadronically decaying  $W^-$ .<sup>2</sup> For the case of a  $ZZ$  decay, the subsequent decays can either be into a pair of muons and a pair of electrons (`nproc=116`), a pair of muons and neutrinos (`nproc=117`) or a pair of muons and a pair of bottom quarks (`nproc=118`).

At LO the relevant diagram is the coupling of two gluons to the Higgs via a top quark loop. This calculation is performed in the limit of infinite top quark mass, so that the top quark loop is replaced by an effective operator. This corresponds to the effective Lagrangian,

$$\mathcal{L} = \frac{1}{12\pi v} G_{\mu\nu}^a G_a^{\mu\nu} H, \quad (12)$$

where  $v$  is the Higgs vacuum expectation value and  $G_{\mu\nu}^a$  the gluon field strength tensor. The calculation may be performed at NLO, although radiation from the bottom quarks in the decay of processes 111 and 118 is not yet included.

When `removebr` is true in processes 111, 112, 113, 118, the Higgs boson does not decay.

Process 119 implements the decay of the Higgs boson into two photons via loops of top quarks and  $W$ -bosons. The decay is implemented using the formula Eq.(11.12) from ref. [34]. When `removebr` is true in process 119 the Higgs boson does not decay.

Processes 120 and 121 implement the decay of the Higgs boson into an lepton-antilepton pair and a photon. As usual the production of a charged lepton-antilepton pair is mediated by a  $Z/\gamma^*$  (process 120) and the production of three types of neutrinos  $\sum \nu\bar{\nu}$  by a  $Z$ -boson (process 121). These processes are implemented using a generalization of the formula of [35]. (Generalization to take into account off-shell  $Z$ -boson and adjustment of the sign of  $C_2$  in their Eq.(4)).

## 6.26. $H \rightarrow W^+W^-$ production, processes 123-126

These processes represent the production of a Higgs boson that decays to  $W^+W^-$ , with subsequent decay into leptons. For process 123, the exact form of the triangle loop coupling a Higgs boson to two gluons is included, with both top and bottom quarks circulating in the loop. This is to be contrasted with process 113 in which only the top quark contribution is included in the effective coupling approach.

Process 124 includes only the effect of the interference of the Higgs and  $gg \rightarrow W^+W^-$  amplitudes, as described in ref. [36]. The calculation is available at LO only. LO corresponds to  $O(\alpha_s^2)$  in this case. The calculation of loops containing the third quark generation includes the effect of the top quark mass (but  $m_b = 0$ ), while the first two generations are considered massless. For numerical stability, a small cut on the transverse momentum of the  $W$  bosons is applied:  $p_T(W) > 0.05$  GeV for loops containing massless (first or second generation) quarks,  $p_T(W) > 2$  GeV for  $(t, b)$  loops. This typically removes less than 0.1% of the cross section. The values of these cutoffs can be changed by editing `src/HWW/gg_WW_int.f` and recompiling.

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<sup>2</sup>We have not included the case of a hadronically decaying  $W^+$ ; it can be obtained from processes `nproc=114,115` by performing the substitutions  $\nu \rightarrow e^-$  and  $e^+ \rightarrow \bar{\nu}$ .

Process 125 includes all  $gg$ -initiated diagrams that have a Higgs boson in the  $s$ -channel, namely the square of the  $s$ -channel Higgs boson production and the interference with the diagrams that do not contain a Higgs boson, (i.e.  $gg \rightarrow W^+W^- \rightarrow \nu_e e^+ e^- \bar{\nu}_e$ ).

The result for the square of the box diagrams alone, i.e. the process  $gg \rightarrow W^+W^- \rightarrow \nu_e e^+ e^- \bar{\nu}_e$ , may be obtained by running process `nproc=61` with `part=virt` and `ggonly=.true.`

Process 126 calculates the full result for this process from  $gg$ -initiated diagrams. This includes diagrams that have a Higgs boson in the  $s$ -channel, the continuum  $W^+W^-$  diagrams described above and their interference.

## 6.27. $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$ production, processes 128-133

These processes represent the production of a Higgs boson that decays to  $ZZ$ , with subsequent decay into charged leptons. For process 128, the exact form of the triangle loop coupling a Higgs boson to two gluons is included, with both top and bottom quarks circulating in the loop. This is to be contrasted with process 116 in which only the top quark contribution is included in the effective coupling approach.

Process 129 includes only the effect of the interference of the Higgs and  $gg \rightarrow ZZ$  amplitudes. The calculation is available at LO only. LO corresponds to  $O(\alpha_s^2)$  in this case. The calculation of loops containing the third quark generation includes the effect of both the top quark mass and the bottom quark, while the first two generations are considered massless. For numerical stability, a small cut on the transverse momentum of the  $Z$  bosons is applied:  $p_T(Z) > 0.05$  GeV. This typically removes less than 0.1% of the cross section. The values of these cutoffs can be changed by editing `src/ZZ/getggZZamps.f` and recompiling.

Process 130 includes all  $gg$ -initiated diagrams that have a Higgs boson in the  $s$ -channel, namely the square of the  $s$ -channel Higgs boson production and the interference with the diagrams that do not contain a Higgs boson, (i.e.  $gg \rightarrow Z/\gamma^* + Z/\gamma^* \rightarrow e^-e^+\mu^-\mu^+$ ).

Process 131 calculates the full result for this process from  $gg$ -initiated diagrams. This includes diagrams that have a Higgs boson in the  $s$ -channel, the continuum  $Z/\gamma^* + Z/\gamma^*$  diagrams described above and their interference.

Process 132 gives the result for the square of the box diagrams alone, i.e. the process  $gg \rightarrow Z/\gamma^* + Z/\gamma^* \rightarrow e^-e^+\mu^-\mu^+$ .

Process 133 calculates the interference for the  $qg$  initiated process.

For those processes that include contributions from the Higgs boson, the form of the Higgs propagator may be changed by editing the file `src/Need/sethparams.f`. If the logical variable `CPscheme` is changed from the default value `.false.` to `.true.` then the Higgs propagator is computed using the “bar-scheme” that is implemented in the HTO code of G. Passarino [37, 38]. The value of the Higgs boson width has been computed with v1.1 of the HTO code, for Higgs masses in the interval  $50 < m_H < 1500$  GeV. These values are tabulated, in 0.5 GeV increments, in the file `Bin/hto_output.dat`. The widths for other masses in this range are obtained by linear interpolation.

### 6.27.1. Specifying other final states

As described above, these processes refer to a final state  $e^-e^+\mu^-\mu^+$ . It is however possible to specify a final state that corresponds to a different set of  $Z$  boson decays. This is achieved by altering the value of `NPROC` in the input file by appending a period, followed by two 2-character strings that identify each of the decays. Possible values for the strings, and the corresponding decays, are shown in the table below.

string	$Z$ decay
e1,EL	$(e^-, e^+)$
mu,MU,m1,ML	$(\mu^-, \mu^+)$
t1,TL	$(\tau^-, \tau^+)$
nu,NU,n1,NL	$(\nu, \bar{\nu}) \times 3$
bq,BQ	$(b, \bar{b})$

Note that, for the case of neutrino decays, the sum over three flavours of neutrino is performed. The labelling of the particles in the output is best understood by example. Setting `nproc=132.ELNU` corresponds to the process  $gg \rightarrow Z/\gamma^* + Z/\gamma^* \rightarrow e^-(p_3)e^+(p_4)\nu(p_5)\bar{\nu}(p_6)$ . Note that the default process corresponds to the string `ELMU` so that, for instance `nproc=132.ELMU` is entirely equivalent to `nproc=132`. The effect of changing the lepton flavour is only seen in the output of LHE events, where the correct mass is then used when producing the event record.

### 6.28. $e^-e^+\nu_e\bar{\nu}_e$ production, processes 1281, 1311, 1321

These processes compute cross sections relevant for the final state  $e^-e^+\nu_e\bar{\nu}_e$ , i.e. with charged leptons and neutrinos from the same (electron) doublet. As a result they receive contributions from diagrams with resonant  $ZZ$  propagators and resonant  $WW$  propagators. Process 1281 only includes amplitudes containing a Higgs boson (c.f. processes 123 and 128). Process 1321 only includes continuum (box-diagram) amplitudes (c.f. processes 127 and 132). Process 1311 includes both amplitudes and the effects of the interference between them (c.f. processes 126 and 131). The effect of the interference between the  $WW$  and  $ZZ$  diagrams can be assessed by, for instance, comparing process 1281 with the sum of processes 123 and one-third of 128.ELNU, where the weighting is to divide out the natural sum over three neutrino flavours in process 128.ELNU.

Event generation is not available for these processes at present.

### 6.29. $e^-e^+\nu\bar{\nu}$ production, processes 1282, 1312, 1322

These processes compute cross sections relevant for the final state  $e^-e^+\nu\bar{\nu}$ , i.e. an electron pair and a sum over all three flavours of neutrino. For muon and tau neutrinos, only  $ZZ$  diagrams contribute. For electron neutrinos there are contributions from diagrams with resonant  $ZZ$  propagators and resonant  $WW$  propagators. Process 1282 only includes amplitudes containing a Higgs boson (c.f. processes 123 and 128). Process 1322 only includes continuum (box-diagram) amplitudes (c.f. processes 127 and 132). Process 1312 includes both amplitudes and the effects of the interference between them (c.f. processes 126 and 131)s. The effect of the

interference between the  $WW$  and  $ZZ$  diagrams can be assessed by, for instance, comparing process 1282 with the sum of processes 123 and 128.ELNU.

Event generation is not available for these processes at present.

### 6.30. $H + b$ production, processes 136–138

[For more details on this calculation, please refer to [hep-ph/0204093](#)]

These processes represent the production of a Standard Model Higgs boson that decays into a pair of bottom quarks, in association with a further bottom quark. The initial state at lowest order is a bottom quark and a gluon. The calculation may be performed at NLO, although radiation from the bottom quarks in the Higgs decay is not included.

For this process, the matrix elements are divided up into a number of different sub-processes, so the user must sum over these after performing more runs than usual. At lowest order one can proceed as normal, using `nproc=136`. For a NLO calculation, the sequence of runs is as follows:

- Run `nproc=136` with `part=virt` and `part=real` (or, both at the same time using `part=tota`);
- Run `nproc=137` with `part=real`.

The sum of these yields the cross-section with one identified  $b$ -quark in the final state. To calculate the contribution with two  $b$ -quarks in the final state, one should use `nproc=138` with `part=real`.

When `removebr` is true, the Higgs boson does not decay.

### 6.31. $t\bar{t}$ production with 2 semi-leptonic decays, processes 141–145

These processes describe  $t\bar{t}$  production including semi-leptonic decays for both the top and the anti-top. In version 6.2 we have updated this to use the one-loop amplitudes of ref. [39]. The code for the virtual amplitudes now runs about three times faster than earlier versions where the virtual amplitudes of ref. [40] were used. Switching `zerowidth` from `.true.` to `.false.` only affects the  $W$  bosons from the top quark decay, because our method of including spin correlations requires the top quark to be on shell.

Process 141 includes all corrections, i.e. both radiative corrections to the decay and to the production. This process is therefore the basic process for the description of top production where both quarks decay semi-leptonically. When `removebr` is true in process 141, the top quarks do not decay. When one wishes to calculate observables related to the decay of the top quark, `removebr` should be false in process 141. The LO calculation proceeds as normal. At NLO, there are two options:

- `part=virt, real` or `tota` : final state radiation is included in the production stage only



- `part = todk` : radiation is included in the decay of the top quark also and the final result corresponds to the sum of real and virtual diagrams. Note that these runs automatically perform an extra integration, so will take a little longer.

Process 142 includes only the corrections in the semileptonic decay of the top quark. Thus it is of primary interest for theoretical studies rather than for physics applications. Because of the method that we have used to include the radiation in the decay, the inclusion of the corrections in the decay does not change the total cross section. This feature is explained in section 6 of ref. [41].

In the case of process 145, there are no spin correlations in the decay of the top quarks. The calculation is performed by multiplying the spin summed top production cross section, by the decay matrix element for the decay of the  $t$  and the  $\bar{t}$ . These processes may be used as a diagnostic test for the importance of the spin correlation.

### 6.32. $t\bar{t}$ production with decay and a gluon, process 143

This process describes lowest order  $t\bar{t} + g$  production including two leptonic decays  $t \rightarrow b\nu$ . When `removebr` is true, the top quarks do not decay. This LO process only includes radiation in production.

### 6.33. $t\bar{t}$ production with one hadronic decay, processes 146–151

These processes describe the hadronic production of a pair of top quarks, with one quark decaying hadronically and one quark decaying semileptonically. For processes 146-148, the top quark decays semileptonically whereas the anti-top quark decays hadronically. For processes 149-151, the top quark decays hadronically whereas the anti-top quark decays semileptonically. The base processes for physics are process 146 and 149 which include radiative corrections in both production and decay. Switching `zerowidth` from `.true.` to `.false.` only affects the  $W$  bosons from the top quark decay, because our method of including spin correlations requires the top quark to be on shell. When one wishes to calculate observables related to the decay of the top quark, `removebr` should be false in processes 146 and 149. The LO calculation proceeds as normal. At NLO, there are two options:

- `part=virt, real` or `tota` : final state radiation is included in the production stage only
- `part = todk` : radiation is included in the decay of the top quark also and the final result corresponds to the sum of real and virtual diagrams. Note that these runs automatically perform an extra integration, so will take a little longer.

Processes 147 and 150 include only the radiative corrections in the decay of the top quark without including the radiative corrections in the hadronic decay of the  $W$ -boson. Because of the method that we have used to include the radiation in the decays, the inclusion of the corrections in this stage of the decay does not change the total cross section. Process 148 (151) includes only the radiative corrections in the hadronic decay of the  $W$ -boson coming from the anti-top (top). The inclusion of the corrections in this stage of the decay increases the partial width by the normal  $\alpha_s/\pi$  factor.

### 6.34. $Q\bar{Q}$ production, processes 157–159

These processes calculate the production of heavy quarks (157 for top, 158 for bottom and 159 for charm) up to NLO using the matrix elements of ref. [42]. No decays are included.

### 6.35. $t\bar{t}+$ jet production, process 160

This process calculates the production of top quarks and a single jet at LO, without any decay of the top quarks.

### 6.36. Single top production, processes 161–177

*[For more details on this calculation, please refer to hep-ph/0408158]*

These processes represent single top production and may be calculated up to NLO as described below.

Single top production is divided as usual into  $s$ -channel (processes 171-177) and  $t$ -channel (161-167) diagrams. Each channel includes separately the production of a top and anti-top quark, which is necessary when calculating rates at the LHC. Below we illustrate the different use of these processes by considering  $t$ -channel top production (161,162), although the procedure is the same for anti-top production (166,167) and the corresponding  $s$ -channel processes (171,172) and (176,177).

To calculate cross-sections that do not include any decay of the (anti-)top quark, one should use process 161 (or, correspondingly, 166, 171 and 176) with `removebr` true. The procedure is exactly the same as for any other process. Switching `zerowidth` from `.true.` to `.false.` only affects the  $W$  boson from the top quark decay.

For processes 161, 162, 163, 166, 167 and 168 the default behaviour when `removebr` is true is that partons are clustered into jets using the supplied jet algorithm parameters, but no cut is applied on the number of jets. This behaviour can be altered by changing the value of the variable `notag` in the file `src/User/setnotag.f`.

When one wishes to calculate observables related to the decay of the top quark, `removebr` should be false. The LO calculation proceeds as normal. At NLO, there are two options:

- `part=virt`, `real` or `tota` : final state radiation is included in the production stage only
- `part = todk` : radiation is included in the decay of the top quark also and the final result corresponds to the sum of real and virtual diagrams. This process can only be performed at NLO with `zerowidth = .true.` This should be set automatically. Note that these runs automatically perform an extra integration, so will take a little longer.

The contribution from radiation in the decay may be calculated separately using process 162. This process number can be used with `part=virt,real` only. To ensure consistency, it is far simpler to use 161 and this is the recommended approach.

A further option is provided for the  $t$ -channel single top process (when no top quark decay is considered), relating to NLO real radiation diagrams that contain a bottom quark. In the processes above the bottom quark is taken to be massless. To include the effect of  $m_b > 0$ , one can run process 163 (168) in place of 161 (166) and additionally include process 231 (236) at leading order. The non-zero bottom quark mass has little effect on the total cross section, but enables a (LO) study of the bottom quark kinematics. Higher order corrections to the bottom quark kinematics can only be studied by running process 231 (236) at NLO.

### 6.37. Off-shell single top production in SM and SMEFT, processes 164,169

*[For more details on this calculation, please refer to ref. [28]]*

The processes 164 and 169 represent off-shell single-top-quark and anti-top-quark production in the complex-mass scheme, respectively. Both the SM and contributions from the SMEFT can be calculated.

Dynamical double deep inelastic scattering scales can be consistently used at NLO by setting `dynamicsscale` to ‘DDIS’ and `scale=facscale` to `1d0`. In this way the momentum transfer along the  $W$ -boson  $Q^2$  is used as the scale for the light-quark-line corrections  $\mu^2 = Q^2$ , and  $\mu^2 = Q^2 + m_t^2$  for the heavy-quark-line corrections. These scales are also consistently used for the non-resonant contributions, with QCD corrections on the  $ud$ -quark line, and separate QCD corrections on the bottom-quark line.

The new block ‘Single top SMEFT, nproc=164,169’ in the input file governs the inclusion of SMEFT operators and corresponding orders. The scale of new physics  $\Lambda$  can be separately set, and has a default value of 1000 GeV. The flag `enable 1/lambda4` enables the  $1/\Lambda^4$  contributions, where operators  $Q_{\varphi ud}^{33}$ ,  $Q_{dW}^{33}$ ,  $Q_{dG}^{33}$ , and  $Q_{4R}$  can contribute for the first time. For the non-Hermitian operators we allow complex Wilson coefficients. We also have a flag to disable the pure SM contribution, leaving only contributions from SMEFT operators either interfered with the SM amplitudes or as squared contributions at  $1/\Lambda^4$ . This can be used to directly and quickly extract kinematical distributions and the magnitudes of pure SMEFT contributions.

To allow for easier comparisons with previous anomalous couplings results, and possibly estimate further higher order effects, we allow for an anomalous couplings mode at LO by enabling the corresponding flag. The relations between our operators and the anomalous couplings are

$$\begin{aligned}\delta V_L &= C_{\varphi q}^{(3,33)} \frac{m_t^2}{\Lambda^2}, \text{ where } V_L = 1 + \delta V_L, \\ V_R &= C_{\varphi ud}^{33} \frac{m_t^2}{\Lambda^2}, \\ g_L &= -4 \frac{m_W m_t}{\Lambda^2} \cdot C_{dW}^{33}, \\ g_R &= -4 \frac{m_W m_t}{\Lambda^2} \cdot C_{uW}^{33},\end{aligned}$$

where  $m_W$  is the  $W$ -boson mass, and  $m_W = \frac{1}{2}g_W v$  has been used to derive this equivalence. Note that the minus sign for  $g_L$  and  $g_R$  is different from the literature. See also the publication for more information.

For comparisons with on-shell results one needs to add up the contributions from processes 161 at NLO and from the virt and real contributions from 162, see above.

The analysis/plotting routine is contained in the file ‘`src/User/nplotter_ktopanom.f`’, where all observables presented in this study are implemented, and the  $W$ -boson/neutrino reconstruction is implemented and can be switched on or off.

### 6.38. $Wt$ production, processes 180–187

[For more details on this calculation, please refer to *hep-ph/0506289*]

These processes represent the production of a  $W$  boson that decays leptonically in association with a top quark. The lowest order diagram involves a gluon and a bottom quark from the PDF, with the  $b$ -quark radiating a  $W$  boson and becoming a top quark. The calculation can be performed up to NLO.

Processes 180 and 185 produce a top quark that does not decay, whilst in processes 181 and 186 the top quark decays leptonically. Consistency with the simpler processes (180, 185) can be demonstrated by running process 181, 186 with `removebr` set to true.

At next-to-leading order, the calculation includes contributions from diagrams with two gluons in the initial state,  $gg \rightarrow Wtb$ . The  $p_T$  of the additional  $b$  quark is vetoed according to the value of the parameter `ptmin_bjet` which is specified in the input file. The contribution from these diagrams when the  $p_T$  of the  $b$  quark is above `ptmin_bjet` is zero. The values of this parameter and the factorization scale (`facscale`) set in the input file should be chosen carefully. Appropriate values for both (in the range 30-100 GeV) are discussed in the associated paper.

When one wishes to calculate observables related to the decay of the top quark, `removebr` should be false. The LO calculation proceeds as normal. At NLO, there are two options:

- `part=virt, real` or `tota` : final state radiation is included in the production stage only

- `part = todk` : radiation is included in the decay of the top quark also and the final result corresponds to the sum of real and virtual diagrams. This process can only be performed at NLO with `zerowidth = .true`. This should be set automatically. Note that these runs automatically perform an extra integration, so will take a little longer.

The contribution from radiation in the decay may be calculated separately using processes 182,187. These process numbers can be used with `part=virt,real` only. To ensure consistency, it is far simpler to use 181,186 and this is the recommended approach.

### 6.39. $H+$ jet production with finite top-mass effects, process 200

This process represents the production of a Higgs boson in association with a single jet based on refs. [43, 44]. Decay modes are currently unsupported/untested. The top-quark mass is taken into account exactly for the born and real-emission parts, as well as for the singular part of the virtual corrections.

The finite part of the two-loop virtual corrections can be computed in different ways.

- In a low energy asymptotic expansion in  $1/m_t^k$  up to order  $k = 2, 4$  by setting `mtex` to 2 or 4 in the input file. This is recommended for transverse momenta up to  $\simeq 225$  GeV.
- In a high energy expansion by setting `mtex=100` in the input file. This is recommended for transverse momenta beyond 450 GeV.
- In a rescaling approach where the finite part of the two-loop virtual amplitude in the effective field theory ( $m_t = \infty$ ) is rescaled pointwise by the ratio of the one-loop amplitude computed with full  $m_t$  dependence to the one-loop amplitude for  $m_t = \infty$ . This mode is the default mode and enabled with `mtex=0` in the input file. This is the recommended approach for the intermediate energy region and for estimating top-mass uncertainties in the transition regions between these approaches.

### 6.40. Di-jet production, processes 190–191

Process 190 represents di-jet production through strong interactions. It may be calculated to LO only.

Process 191 is an ancillary process that is used in the calculation of weak one-loop corrections to di-jet production. When computed at LO it gives the contribution of weak ( $\mathcal{O}(\alpha^2)$ ) and mixed weak-strong ( $\mathcal{O}(\alpha\alpha_s)$ ) mediated processes to di-jet production. Please refer to Ref. [22] for details.

### 6.41. $H+$ jet production, processes 201–210

These processes represent the production of a Higgs boson in association with a single jet, with the subsequent decay of the Higgs to either a pair of bottom quarks (processes 201,203,206) or to a pair of tau's (202,204,207), or to a pair of  $W$ 's which decay leptonically (208), or to a pair of  $Z$ 's which decay leptonically (209), or to a pair of photons (210).

The Higgs boson couples to a pair of gluons via a loop of heavy fermions which, in the Standard Model, is accounted for almost entirely by including the effect of the top quark alone. For processes 201, 202, 206, 207, the matrix elements include the full dependence on the top quark mass. The calculation can only be performed at LO. However, the Higgs boson can either be the Standard Model one (processes 201, 202) or a pseudoscalar (206, 207). Note that the pseudoscalar case corresponds, in the heavy top limit, to the effective Lagrangian,

$$\mathcal{L} = \frac{1}{8\pi v} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} A, \quad (13)$$

where  $\tilde{G}_a^{\mu\nu} = i\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$ . The interaction differs from the scalar case in Eq. 12 by a factor of  $3/2$  and hence the rate is increased by a factor of  $(3/2)^2$ .

For processes 203, 204, 208, 209, 210, the calculation is performed in the limit of infinite top quark mass, so that NLO results can be obtained. The virtual matrix elements have been implemented from refs [45] and [46]. Phenomenological results have previously been given in refs. [47],[45] and [48]. Note that the effect of radiation from the bottom quarks in process 203 is not included.

When `removebr` is true in processes 201, 203, 206, 208, 209 and 210, the Higgs boson does not decay.

## 6.42. Higgs production via WBF, processes 211–217

*[For more details on this calculation, please refer to hep-ph/0403194]*

These processes provide predictions for the production of a Higgs boson in association with two jets via weak-boson fusion (WBF). The Higgs boson subsequently decays to either a pair of bottom quarks (processes 211, 216), to a pair of tau's (212, 217), to a pair of  $W$  bosons (213), to a pair of  $Z$  bosons (214), or to a pair of photons (215).

Calculations can be performed up to NLO for processes 211–215. In addition to this, processes 216 and 217 provide the lowest order calculation of the WBF reaction which radiates an additional jet.

When `removebr` is true, the Higgs boson does not decay.

## 6.43. $\tau^+\tau^-$ production, process 221

This process provides predictions for the production of a tau lepton pair mediated by  $\gamma^*/Z$ , with subsequent leptonic decays. The calculation is available at LO only. The relevant matrix elements are adapted from the ones in ref. [49].

When `removebr` is true, the tau leptons do not decay.

#### 6.44. $e^-e^+\nu_\mu\bar{\nu}_\mu$ -pair production via WBF, processes 222

The *weak* processes occur in  $O(\alpha^6)$ , whereas the *strong* processes occur in  $O(\alpha^4\alpha_S^2)$ . These processes can currently only be calculated at lowest order.

$$\begin{aligned} 222 \quad & f(p_1) + f(p_2) \rightarrow Z(e^-(p_3), e^+(p_4))Z(\nu_\mu(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)[WBF] \\ 2201 \quad & f(p_1) + f(p_2) \rightarrow Z(e^-(p_3), e^+(p_4))Z(\mu^-(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[strong] \\ 2221 \quad & f(p_1) + f(p_2) \rightarrow Z(e^-(p_3), e^+(p_4))Z(\nu_\mu(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)[strong] \end{aligned}$$

#### 6.45. $\nu_e e^+ \mu^- \mu^+$ -pair production via WBF, processes 223,2231

The *weak* processes occur in  $O(\alpha^6)$ , whereas the *strong* processes occur in  $O(\alpha^4\alpha_S^2)$ . These processes can currently only be calculated at lowest order.

$$\begin{aligned} 223 \quad & f(p_1) + f(p_2) \rightarrow W^+(\nu_e(p_3), e^+(p_4))Z(\mu^-(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[weak] \\ 2231 \quad & f(p_1) + f(p_2) \rightarrow W^+(\nu_e(p_3), e^+(p_4))Z(\mu^-(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[strong] \end{aligned}$$

#### 6.46. $e^- \bar{\nu}_e \nu_\mu \mu^+$ -pair production via WBF, processes 224,2241

The *weak* processes occur in  $O(\alpha^6)$ , whereas the *strong* processes occur in  $O(\alpha^4\alpha_S^2)$ . These processes can currently only be calculated at lowest order.

$$\begin{aligned} 224 \quad & f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[WBF] \\ 2241 \quad & f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[strong] \end{aligned}$$

#### 6.47. $e^- \bar{\nu}_e \mu^- \mu^+$ -pair production via WBF, processes 225,2251

The *weak* processes occur in  $O(\alpha^6)$ , whereas the *strong* processes occur in  $O(\alpha^4\alpha_S^2)$ . These processes can currently only be calculated at lowest order.

$$\begin{aligned} 225 \quad & f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))Z(\mu^-(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[weak] \\ 2251 \quad & f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))Z(\mu^-(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[strong] \end{aligned}$$

#### 6.48. $e^- e^+ \bar{\nu}_e \nu_e$ -pair production via WBF, processes 226

The *weak* processes occur in  $O(\alpha^6)$ , whereas the *strong* processes occur in  $O(\alpha^4\alpha_S^2)$ . This process can currently only be calculated at lowest order.

$$226 \quad f(p_1) + f(p_2) \rightarrow e^-(p_3) + e^+(p_4) + \nu_e(p_5) + \bar{\nu}_e(p_6) + f(p_7) + f(p_8)[WBF]$$

#### 6.49. $\nu_e e^+ \nu_\mu \mu^+$ -pair production via WBF, processes 228,2281

This is pure electroweak process, occurring in  $O(\alpha^6)$ . These processes can currently only be calculated at lowest order.

$$\begin{aligned} 228 \quad & f(p_1) + f(p_2) \rightarrow W^+(\nu_e(p_3), e^+(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[WBF] \\ 2281 \quad & f(p_1) + f(p_2) \rightarrow W^+(\nu_e(p_3), e^+(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)[strong] \end{aligned}$$

## 6.50. $e^- \bar{\nu}_e \mu^- \bar{\nu}_\mu$ -pair production via WBF, processes 229,2291

This is pure electroweak process, occurring in  $O(\alpha^6)$ . These processes can currently only be calculated at lowest order.

$$229 \quad f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))W^+(\mu^-(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)[WBF]$$

$$2291 \quad f(p_1) + f(p_2) \rightarrow W^-(e^-(p_3), \bar{\nu}_e(p_4))W^-(\mu^-(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)[strong]$$

## 6.51. $t$ -channel single top with an explicit $b$ -quark, processes 231–240

[For more details on this calculation, please refer to *arXiv:0903.0005 [hep-ph]*]

These represent calculations of the  $t$ -channel single top (231) and anti-top (231) processes in a scheme with four flavours of quark in the proton, so that  $b$ -quarks are not present in the proton. The  $b$ -quark is instead explicitly included in the final state.

Processes 232 and 236 represent  $t$ -channel single top production in association with a further jet and may be calculated at LO only.

Processes 233 and 238 are the complete four-flavour scheme  $t$ -channel single top production processes. These are therefore the processes that should be used for most physics applications. When one wishes to calculate observables related to the decay of the top quark, `removebr` should be false in processes 233 and 236. The LO calculation proceeds as normal. At NLO, there are two options:

- `part=virt`, `real` or `tota` : final state radiation is included in the production stage only
- `part = todk` : radiation is included in the decay of the top quark also and the final result corresponds to the sum of real and virtual diagrams. Note that these runs automatically perform an extra integration, so will take a little longer.

Processes 234 and 239 give the extra contribution due to radiation in top decay. These processes are mainly of theoretical interest.

Processes 235 and 240 are the leading order single top processes with an extra radiated parton. These processes do not include jets produced in the decay process.

## 6.52. $W^+W^+$ +jets production, processes 251,252

These processes represent the production of two  $W^+$  bosons in association with two (process 251) or three (process 252) jets. The lowest order at which two positively charged  $W$  bosons can be produced is with two jets. This process is only implemented for leptonic decays of the  $W$  particles. The calculation is available at LO only. The calculation and code are from ref. [50]. `removebr` is not implemented and has no effect.



### 6.53. $Z + Q$ production, processes 261–267

[For more details on this calculation, please refer to [hep-ph/0312024](#)]

These processes represent the production of a  $Z$  boson that decays into a pair of electrons, in association with a heavy quark,  $Q$ .

For processes 261, 262, 266 and 267 the initial state at lowest order is the heavy quark and a gluon and the calculation may be performed at NLO. As for  $H + b$  production, the matrix elements are divided into two sub-processes at NLO. Thus the user must sum over these after performing more runs than usual. At lowest order one can proceed as normal, using `nproc=261` (for  $Z + b$ ) or `nproc=262` (for  $Z + c$ ). For a NLO calculation, the sequence of runs is as follows:

- Run `nproc=261` (or 262) with `part=virt` and `part=real` (or, both at the same time using `part=tota`);
- Run `nproc=266` (or 267) with `part=real`.

The sum of these yields the cross-section with one identified heavy quark in the final state when `inclusive` is set to `.false.`. To calculate the rate for at least one heavy quark, `inclusive` should be `.true.`.

For processes 263 and 264, the calculation uses the matrix elements for the production of a  $Z$  and a heavy quark pair and demands that one of the heavy quarks is not observed. It may either lie outside the range of  $p_T$  and  $\eta$  required for a jet, or both quarks may be contained in the same jet. Due to the extra complexity (the calculation must retain the full dependence on the heavy quark mass), this can only be computed at LO.

When `removebr` is true, the  $Z$  boson does not decay.

### 6.54. $H + 2$ jet production, processes 270–274

These processes represent the production of a Standard Model Higgs boson in association with two jets. The Higgs boson subsequently decays to either a pair of photons (`nproc=270`), a bottom quark pair (`nproc=271`), a pair of tau's (`nproc=272`), a pair of leptonically decaying  $W$ 's (`nproc=273`) or a pair of leptonically decaying  $Z$ 's (`nproc=274`).

The matrix elements are included in the infinite top mass limit using the effective Lagrangian approach.

When `removebr` is true, the Higgs boson does not decay.

### 6.55. $H + 3$ jet production, processes 275–278

These processes represent the production of a Standard Model Higgs boson in association with three jets. The Higgs boson subsequently decays to either a bottom quark pair (`nproc=275`), or a pair of tau's (`nproc=276`) or a pair of  $W$ 's that decay leptonically into a single generation of leptons (`nproc=278`) or a pair of  $Z$ 's that decay leptonically into a single generation of leptons (`nproc=279`). The matrix elements are included in the infinite top mass limit using an effective Lagrangian approach. These calculations can be performed at LO only.

When `removebr` is true, the Higgs boson does not decay.

### 6.56. Direct $\gamma$ production, processes 280–282

These processes represent the production a real photon. Since this process includes a real photon, the cross section diverges when the photon is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. This will ensure that the cross section is well-defined.

The calculation of process 282 is only available at leading order.

### 6.57. Direct $\gamma +$ heavy flavour production, processes 283–284

These processes represent the production a real photon with a  $b$  quark or a charm quark. Since this process includes a real photon, the cross section diverges when the photon is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. This will ensure that the cross section is well-defined.

The calculation of process 283–284 is only available at leading order.

### 6.58. $\gamma\gamma$ production, processes 285–286

Process 285 represents the production of a pair of real photons. Since this process includes two real photons, the cross section diverges when one of the photons is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. This will ensure that the cross section is well-defined.

The calculation of process 285 may be performed at NLO using either the Frixione algorithm [24] or standard cone isolation. This process also includes the one-loop gluon-gluon contribution as given in ref. [51]. The production of a photon via parton fragmentation is included at NLO and can be run separately by using the `frag` option in `part`. This option includes the contributions from the integrated photon dipole subtraction terms and the LO QCD matrix element multiplied by the fragmentation function.

The phase space cuts for the final state photons are defined in `input.ini`, for multiple photon processes such as 285 - 287 the  $p_T$ 's of the individual photons (hardest, second hardest and third hardest or softer) can be controlled independently. The remaining cuts on  $R_{\gamma j}$ ,  $\eta_\gamma$  etc.

are applied universally to all photons. Users wishing to alter this feature should edit the file `photon_cuts.f` in the directory `src/User`.

Process 286, corresponding to  $\gamma\gamma$ +jet production, can be computed at leading order only.

### 6.59. $\gamma\gamma\gamma$ production, process 287

Process 287 represents the production of three real photons. The cross section diverges when one of the photons is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. This will ensure that the cross section is well-defined.

The calculation of process 285 may be performed at NLO using either the Frixione algorithm [24] or standard cone isolation. The production of a photon via parton fragmentation is included at NLO and can be run separately by using the `frag` option in `part`. This option includes the contributions from the integrated photon dipole subtraction terms and the LO QCD matrix element multiplied by the fragmentation function. The phase space cuts for the final state photons are defined in `input.ini`, for multiple photon processes such as 285 - 287 the  $p_T$ 's of the individual photons (hardest, next-to hardest and softest) can be controlled independently. The remaining cut on  $R_{\gamma j}$ ,  $\eta_\gamma$  etc. are applied universally to all photons. Users wishing to alter this feature should edit the file `photon_cuts.f` in the directory `src/User`.

### 6.60. $\gamma\gamma\gamma\gamma$ production, process 289

Process 289 represents the production of four real photons. The cross section diverges when one of the photons is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. This will ensure that the cross section is well-defined.

The calculation of process 289 may be performed at NLO using either the Frixione algorithm [24] or standard cone isolation. The production of a photon via parton fragmentation is included at NLO and can be run separately by using the `frag` option in `part`. This option includes the contributions from the integrated photon dipole subtraction terms and the LO QCD matrix element multiplied by the fragmentation function. The phase space cuts for the final state photons are defined in `input.ini`, for multiple photon processes such as 285 - 289 the  $p_T$ 's of the individual photons (hardest, next-to hardest and softest) can be controlled independently. The remaining cut on  $R_{\gamma j}$ ,  $\eta_\gamma$  etc. are applied universally to all photons. Users wishing to alter this feature should edit the file `photon_cuts.f` in the directory `src/User`.

Note that for this process the second softest and softest photons are forced to have equal minimum  $p_T$ , defined by the `[ptmin_photon(3rd)]` variable in the input file.

### 6.61. $W\gamma$ production, processes 290-297

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a real photon. Since this process includes a real photon, the cross section diverges when the photon is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon`

and `etamax_photon`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false.` the decay  $W \rightarrow \ell\nu$  will include photon radiation from the lepton, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined. Virtual amplitudes are taken from ref. [32].

The calculation of processes 290 and 295 may be performed at NLO using the Frixione algorithm [24] or standard isolation.

For processes 290 and 295 the role of `mtrans34cut` changes to become a cut on the transverse mass on the  $M_{345}$  system, i.e. the photon is included with the leptons in the cut.

### 6.61.1. Anomalous $WW\gamma$ couplings

Processes 290-297 may also be computed including the effect of anomalous  $WW\gamma$  couplings, making use of the amplitudes calculated in Ref. [52]. Including only dimension 6 operators or less and demanding gauge,  $C$  and  $CP$  invariance gives the general form of the anomalous vertex [52],

$$\begin{aligned} \Gamma_{WW\gamma}^{\alpha\beta\mu}(q, \bar{q}, p) &= \bar{q}^\alpha g^{\beta\mu} \left( 2 + \Delta\kappa^\gamma + \lambda^\gamma \frac{q^2}{M_W^2} \right) - q^\beta g^{\alpha\mu} \left( 2 + \Delta\kappa^\gamma + \lambda^\gamma \frac{\bar{q}^2}{M_W^2} \right) \\ &\quad + (\bar{q}^\mu - q^\mu) \left[ -g^{\alpha\beta} \left( 1 + \frac{1}{2} p^2 \frac{\lambda^\gamma}{M_W^2} \right) + \frac{\lambda^\gamma}{M_W^2} p^\alpha p^\beta \right], \end{aligned} \quad (14)$$

where the overall coupling has been chosen to be  $-|e|$ . The parameters that specify the anomalous couplings,  $\Delta\kappa^\gamma$  and  $\lambda^\gamma$ , are specified in the input file as already discussed in Section 6.22. If the input file contains a negative value for the form-factor scale  $\Lambda$  then no suppression factors are applied to the anomalous couplings. Otherwise, the couplings are included in MCFM only after suppression by dipole form factors,

$$\Delta\kappa^\gamma \rightarrow \frac{\Delta\kappa_1^\gamma}{(1 + \hat{s}/\Lambda^2)^2}, \quad \lambda^\gamma \rightarrow \frac{\Delta\lambda^\gamma}{(1 + \hat{s}/\Lambda^2)^2},$$

where  $\hat{s}$  is the  $W\gamma$  pair invariant mass.

The Standard Model cross section is obtained by setting  $\Delta\kappa^\gamma = \lambda^\gamma = 0$ .

### 6.62. $Z\gamma$ , production, processes 300, 305

Processes 300 and 305 represent the production of a  $Z$  boson (or virtual photon for process 300) in association with a real photon based on ref. [53]. The  $Z/\gamma^*$  subsequently decays into either an  $e^+e^-$  pair (`nproc=300`) or neutrinos (`nproc=305`). Since these processes include a real photon, the cross section diverges when the photon is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false.` the decay  $Z \rightarrow e^-e^+$  (`nproc=300`) will include photon radiation from both leptons, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined. The calculation of processes 300 may be performed at NNLO using the Frixione algorithm [24] or standard isolation.

For the process 300 the role of `mtrans34cut` changes to become a cut on the invariant mass on the  $M_{345}$  system, i.e. the photon is included with the leptons in the cut.

### 6.62.1. Anomalous $ZZ\gamma$ and $Z\gamma\gamma$ couplings

Processes 300-305 may also be computed including the effect of anomalous couplings between  $Z$  bosons and photons. Note that, at present, the effect of anomalous couplings is not included in the gluon-gluon initiated contributions.

The anomalous  $Z\gamma Z$  vertex (not present at all in the Standard Model), considering operators up to dimension 8, is given by [52],

$$\begin{aligned} \Gamma_{Z\gamma Z}^{\alpha\beta\mu}(q_1, q_2, p) = & \frac{i(p^2 - q_1^2)}{M_Z^2} \left( h_1^Z (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\mu\beta}) \right. \\ & \left. + \frac{h_2^Z}{M_Z^2} p^\alpha (p \cdot q_2 g^{\mu\beta} - q_2^\mu p^\beta) - h_3^Z \varepsilon^{\mu\alpha\beta\nu} q_{2\nu} - \frac{h_4^Z}{M_Z^2} \varepsilon^{\mu\beta\nu\sigma} p^\alpha p_\nu q_{2\sigma} \right) \end{aligned} \quad (15)$$

where the overall coupling has been chosen to be  $|e|$  (and  $\epsilon^{0123} = +1$ ). The non-standard  $Z_\alpha(q_1)\gamma_\beta(q_2)\gamma_\mu(p)$  momentum-space vertex can be obtained from this equation by setting  $q_1^2 \rightarrow 0$  and replacing  $h_i^Z \rightarrow h_i^\gamma$ . The parameters that specify the anomalous couplings,  $h_i^Z$  and  $h_i^\gamma$  (for  $i = 1 \dots 4$ ), are specified in the input file as, e.g. `h1(Z)` and `h1(gamma)`. If the input file contains a negative value for the form-factor scale  $\Lambda$  then no suppression factors are applied to these anomalous couplings. Otherwise, the couplings are included in MCFM only after suppression by dipole form factors,

$$h_{1,3}^{Z/\gamma} \rightarrow \frac{h_{1,3}^{Z/\gamma}}{(1 + \hat{s}/\Lambda^2)^3}, \quad h_{2,4}^{Z/\gamma} \rightarrow \frac{h_{2,4}^{Z/\gamma}}{(1 + \hat{s}/\Lambda^2)^4},$$

where  $\hat{s}$  is the  $Z\gamma$  pair invariant mass. Note that these form factors are slightly different from those discussed in Sections 6.22 and 6.61. The form factors can be modified in `src/Need/set_anomcoup.f`.

The Standard Model cross section is obtained by setting  $h_i^Z = h_i^\gamma = 0$  for  $i = 1 \dots 4$ .

### 6.63. $Z\gamma\gamma$ production processes, 301, 306

Processes 301 and 306 represent the production of a  $Z$  boson (or virtual photon for process 301) in association with two photons. The  $Z/\gamma^*$  subsequently decays into either an  $e^+e^-$  pair (`nproc=301`) or neutrinos (`nproc=306`). Since these processes include real photons, the cross section diverges when either of the photons is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false.` the decay  $Z \rightarrow e^-e^+$  (`nproc=301`) will include photon radiation from both leptons, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined. Anomalous couplings are not currently implemented for these processes.

## 6.64. $Z\gamma j$ , production, processes 302, 307

Processes 302 and 307 represent the production of a  $Z$  boson (or virtual photon) in association with a real photon and at least one jet. The  $Z/\gamma^*$  subsequently decays into either an  $e^+e^-$  pair (`nproc=302`) or neutrinos (`nproc=307`). Since these processes include a real photon and a jet, the cross section diverges when the photon or jet is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`, and `ptjet_min` and `etajet_max`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false`, the decay  $Z \rightarrow e^-e^+$  (`nproc=302`) will include photon radiation from both leptons, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined. The calculation of processes 302 and 307 may be performed at NLO using the Frixione algorithm [24] or standard isolation. Anomalous couplings are not currently implemented for these processes.

## 6.65. $Z\gamma\gamma j$ and $Z\gamma jj$ , 303, 304, 308 and 309

These processes are available at LO only. The  $Z/\gamma^*$  subsequently decays into either an  $e^+e^-$  pair (`nproc=303,304`) or neutrinos (`nproc=308,309`). Since these processes include a real photon and a jet, the cross section diverges when a photon or a jet is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`, and `ptjet_min` and `etajet_max`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false`, the decay  $Z \rightarrow e^-e^+$  (`nproc=303, 304`) will include photon radiation from both leptons, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined. Anomalous couplings are not currently implemented for these processes.

## 6.66. $W + Q + \text{jet}$ production processes 311–326

These processes represent the production of a  $W$  boson that decays leptonically, in association with a heavy quark,  $Q$  and an additional light jet. In processes 311 and 316  $Q$  is a bottom quark, whilst processes 321 and 326 involve a charm quark. In these processes the quark  $Q$  occurs as parton PDF in the initial state. The initial state in these processes consists of a light quark and a heavy quark, with the light quark radiating the  $W$  boson. These calculations may be performed at LO only.

When `removebr` is true, the  $W$  boson does not decay.

## 6.67. $W + c + \text{jet}$ production, processes 331, 336

These processes represent the production of a  $W$  boson that decays leptonically, in association with a charm quark and an additional light jet.

In contrast to processes 321 and 326 described above, the initial state in this case consists of two light quarks, one of which is a strange quark which radiates the  $W$  boson. The calculation may be performed at LO only.

When `removebr` is true, the  $W$  boson does not decay.

## 6.68. $Z + Q$ +jet production, processes 341–357

[For more details on this calculation, please refer to [hep-ph/0510362](#)]

These processes represent the production of a  $Z$  boson that decays into a pair of electrons, in association with a heavy quark,  $Q$  and an untagged jet.

For processes 341 and 351 the initial state at lowest order is the heavy quark and a gluon and the calculation may be performed at NLO. Thus in these processes the quark  $Q$  occurs as parton PDF in the initial state. As for  $H + b$  and  $Z + Q$  production, the matrix elements are divided into two sub-processes at NLO. Thus the user must sum over these after performing more runs than usual. At lowest order one can proceed as normal, using `nproc=341` (for  $Zbj$ ) or `nproc=351` (for  $Zcj$ ). For a NLO calculation, the sequence of runs is as follows:

- Run `nproc=341` (or 351) with `part=virt` and `part=real` (or, both at the same time using `part=tota`);
- Run `nproc=342` (or 352) with `part=real`.

The sum of these yields the cross-section with one identified heavy quark and one untagged jet in the final state when `inclusive` is set to `.false.`. To calculate the rate for at least one heavy quark and one jet (the remaining jet may be a heavy quark, or untagged), `inclusive` should be `.true.`.

Processes 346, 347 and 356, 357 are the lowest order processes that enter the above calculation in the real contribution. They can be computed only at LO.

When `removebr` is true, the  $Z$  boson does not decay.

## 6.69. $c\bar{s} \rightarrow W^+$ , processes 361–363

These processes represent the production of a  $W^+$  from a charm and anti-strange quark at LO. The  $W^+$  boson decays into a neutrino and a positron.

The NLO corrections to this LO process include a contribution of the form,  $g\bar{s} \rightarrow W^+\bar{c}$ . For process 361 this contribution is calculated in the approximation  $m_c = 0$  at NLO. In order to perform the NLO calculation for a non-zero value of  $m_c$ , one must instead sum the results of processes 362 and 363 for `part=tota`.

## 6.70. $W\gamma\gamma$ production, processes 370-371

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with two real photons. Since this process includes real photons, the cross section diverges when the photon is very soft or in the direction of the beam. Thus in order to produce sensible results, the input file must supply values for both `ptmin_photon` and `etamax_photon`. Moreover, when the parameters `zerowidth` and `removebr` are set to `.false.` the decay  $W \rightarrow \ell\nu$  will include photon radiation from the lepton, so that a non-zero `R(photon,lept)_min` should also be supplied. This will ensure that the cross section is well-defined.

These processes may be computed at leading order only.

Process ( $W^+/W^-$ )	<code>inclusive=.false.</code>	<code>inclusive=.true.</code>
401/406	$(b)$ or $(\bar{b})$	$+$ $(b, \bar{b})$ or $(b, j)$ or $(\bar{b}, j)$
402/407	$(B)$	$+$ $(B, j)$
403/408	$(b, \bar{b})$	(no extra configurations)

Table 29: The different final states allowed in the calculation of processes 401–408. A jet containing both  $b$  and  $\bar{b}$  quarks is denoted by  $B$  and a light (quark or gluon) jet by  $j$ . The inclusive (right-hand) column also allows the final states in the exclusive (middle) column.

### 6.71. $W + Q$ production in the 4FNS, processes 401–408

These processes represent the production of a  $W$  boson and one or more jets, at least one of which is a  $b$ -quark, calculated in the 4-flavour number scheme (4FNS). This implies that contributions that explicitly contain a  $b$ -quark in the initial state are not included. These processes all use the same matrix elements as processes 20 and 25 (see section 6.6), but make different cuts on the final state. The final state is specified by the process number and the value of the flag `inclusive`, as shown in Table 29. An additional flag is hard-coded into the file `src/User/filterWbbmas.f` to control the inclusion of the 3-jet configuration,  $(b, \bar{b}, j)$  when `inclusive` is set to `.true.`. By default this is included, `veto3jets = .false.`. If this flag is set to `.true.` then the  $(b, \bar{b}, j)$  contribution is not included in processes 401, 402, 406, 407.

As usual, jets may be unobserved as a result of them falling outside the  $p_T$  and rapidity ranges specified in the input file. In addition, the number of jets may be different from the number of partons in the matrix element calculation as a result of merging in the jet algorithm.

### 6.72. $W + Q$ production in the 5FNS, processes 411, 416

These processes represent production of a  $W$  boson in association with a  $b$ -jet, computed in the 5-flavour number scheme, i.e. a  $b$ -quark is present in the initial state. The lowest order processes are the same as in processes 311, 316. The results at NLO are not physical cross sections since part of the corrections are not included in order to avoid double-counting with the 4FNS process (processes 401 and 406). To obtain combined 4FNS+5FNS predictions, the user should select process 421 ( $W^+$ ) or 426 ( $W^-$ ).

### 6.73. $W + Q$ production in the combined 4FNS/5FNS, processes 421, 426

These processes represent the production of a  $W$  boson and one or more jets, at least one of which is a  $b$ -quark, calculated by combining the 4- and 5-flavour results of processes 401, 411 (for 421) and 406, 416 (for 426). The selection of the final state is the same as for processes 401 and 406, as described in Section 6.71. The procedure for combining the two calculations is described in refs. [54, 55].



## 6.74. $W + b\bar{b}$ jet production, processes 431,436

These processes represent the production of a  $W$  boson which subsequently decays leptonically, in association with a  $b\bar{b}$  pair and an additional jet. The effect of the bottom quark mass is included (c.f. the massless approximation used in processes 24, 29) and the calculation may be performed at LO only.

When `removebr` is true, the  $W$  boson does not decay.

## 6.75. $W + t\bar{t}$ processes 500–516

These processes represent the production of a  $W^\pm$  boson which subsequently decays leptonically, in association with a  $t\bar{t}$  pair. In all except processes 500 and 510 the decays of the top and anti-top quark are included. Processes 501, 502 and 511, 512 refer to the semileptonic decay of the top and antitop quarks, with the latter process in each pair giving the radiation in the decay of the top and antitop. Process 503 (513) refers to the semileptonic decay of the top (antitop) and the hadronic decay of the antitop (top). Processes 506(516) gives the semileptonic decay of the antitop(top) and the hadronic decay of the top(antitop). Processes 506(516) do not give same sign lepton events, so they may be of less phenomenological importance. For this reason we have not yet included radiation in the decay for these processes.

For processes 503, 506, 513 and 516 the default behaviour is that the hadronic decay products are clustered into jets using the supplied jet algorithm parameters, but no cut is applied on the number of jets. This behaviour can be altered by changing the value of the variable `notag` in the file `src/User/setnotag.f`.

The top quarks are always produced on-shell, which is a necessity for a gauge invariant result from this limited set of diagrams, but all spin correlations are included. Switching `zerowidth` from `.true.` to `.false.` only affects the  $W$  bosons (both the directly produced one and from the top quark decay). Processes 501 and 511 may be run at NLO with the option `todk`, including radiation in the decay of the top quark, see section 6.31.

## 6.76. $Zt\bar{t}$ production, processes 529-533

These processes represent the production of a  $Z$  boson in association with a pair of top quarks. For process 529, neither the top quarks nor the  $Z$ -boson decays. In processes 530–533, the top quarks are always produced on-shell, which is a necessity for a gauge invariant result from this limited set of diagrams. Switching `zerowidth` from `.true.` to `.false.` only affects the  $Z$  and the  $W$  bosons from the top quark decay. In process 530 the  $Z$  boson decays into an electron pair, whilst in 531 the decay is into a massless bottom quark pair. In process 532–533 the  $Z$  boson decays into an electron pair, whilst on or other of the top quark or top anti-quark decays hadronically. The calculations can be performed at LO only.

For processes 532 and 533 the default behaviour is that the hadronic decay products are clustered into jets using the supplied jet algorithm parameters, but no cut is applied on the number of jets. This behaviour can be altered by changing the value of the variable `notag` in the file `src/User/setnotag.f`.

When `removebr` is true in process 530, the top quarks and the  $Z$  boson do not decay.

### 6.77. $Ht$ and $H\bar{t}$ production, processes 540–557

[For more details on this calculation, please refer to *arXiv:1302.3856*]

These processes describe the production of a single top quark (540, 544, 550, 554) or antiquark (541, 547, 551, 557) by  $W$  exchange in the  $t$ -channel, in association with a Higgs boson. These processes can be performed at NLO. For processes 540, 541, 550, 551, the top quark does not decay, but the Higgs boson decays to  $b\bar{b}$ , (540, 541), or to  $\gamma\gamma$ , (550, 551). Processes 544, 547 and 554, 557 include the decay of the top quark and antiquark in the approximation in which the top quark is taken to be on shell allowing a clean separation between production and decay.

It is possible to study the effects of anomalous couplings of the Higgs boson to the top quark and  $W$  bosons. These are parametrized by  $c_{t\bar{t}H} = g_{t\bar{t}H}/g_{t\bar{t}H}^{SM}$  and  $c_{WWH} = g_{WWH}/g_{WWH}^{SM}$  respectively, so that  $c_{t\bar{t}H} = c_{WWH} = 1$  in the SM. Different couplings may be chosen by modifying the variables `cttH` and `cWWH` in `src/Need/reader_input.f` and recompiling.

### 6.78. $Zt$ and $Z\bar{t}$ production, processes 560–569

[For more details on this calculation, please refer to *arXiv:1302.3856*]

These processes describe the production of a single top quark (or antiquark) by  $W$  exchange in the  $t$ -channel, in association with a  $Z$  boson. Processes 560, 561, 564, 567 can be performed at NLO. Processes 560-563 are for stable top quarks, whereas processes 564-569 include the decay of the top quark and antiquark in the approximation in which the top quark is taken to be on shell allowing a clean separation between production and decay.

For processes 564 and 567 the default behaviour is that the hadronic decay products are clustered into jets using the supplied jet algorithm parameters, but no cut is applied on the number of jets. This behaviour can be altered by changing the value of the variable `notag` in the file `src/User/setnotag.f`.

### 6.79. $HH$ production, processes 601–602

These processes represent the production of a pair of Higgs bosons. The production proceeds through gluon-fusion one-loop diagrams involving loops of top quarks. The formulae implemented in the code are taken from ref. [56], where the two Higgs bosons are treated as being on-shell. To enforce this condition, the code sets `zerowidth` to true, overriding the value set in the input file. The calculation can be performed at LO only, (i.e. one-loop order only). Two decays of the Higgs bosons are currently foreseen, although other decays can easily be implemented. In process 601, one Higgs boson decays to a pair of  $b$ -quarks, and the other decays to a pair of  $\tau$ 's. In process 602, one Higgs boson decays to a pair of  $b$ -quarks, and the other decays to a pair of photons.

## 6.80. $Ht\bar{t}$ production, processes 640–660

These processes represent the production of a Higgs boson in association with a pair of top quarks. The calculation can be performed at LO only.

For process 640, neither the top quarks nor the Higgs boson decays. In processes 641–647, the top quarks are always produced on-shell, which is a necessity for a gauge invariant result from this limited set of diagrams. Switching `zerowidth` from `.true.` to `.false.` only affects the Higgs and the  $W$  bosons from the top quark decay. In process 641 both the top quarks decay leptonically and the Higgs boson decays into a pair of bottom quarks. Consistency with the simpler process (640) can be demonstrated by running process 641 with `removebr` set to true. In process 644 the top quark decays leptonically and the anti-top quark decays hadronically and the Higgs boson decays into a pair of bottom quarks. In process 647 the anti-top quark decays leptonically and the top quark decays hadronically and the Higgs boson decays into a pair of bottom quarks.

Processes 651–657 correspond to processes 641–647 but with the Higgs decaying to two photons. Processes 661–667 correspond to processes 641–647 but with the Higgs decaying to two  $W$ -bosons which subsequently decay leptonically.

## 6.81. Dark Matter Processes Mono-jet and Mono-photon 800-848

*[For more details on this calculation, please refer to arXiv:1211.6390]*

**This process is currently only officially supported with version 8.0 and earlier, use at your own risk!**

This section provides an overview of the Dark Matter (DM) processes available in MCFM. Since these processes are quite different in the range of possible input parameters (reflecting the range of potential BSM operators) the majority of the new features are controlled by the file `dm_parameters.DAT` located in the `Bin` directory. We begin this section by describing the inputs in this file. Note that these processes are still controlled, as usual by `input.ini` which is responsible for selecting the process, order in perturbation theory, PDFs and phase space cuts etc. The new file controls only the new BSM parameters in the code.

- `[dm mass]` This parameter sets the mass of the dark matter particle  $m_\chi$ .
- `[Lambda]` Controls the mass scale associated with the suppression of the higher dimensional operator in the effective theory approach. Note that each operator has a well defined scaling with respect to Lambda, so cross sections and distributions obtained with one particular value can be readily extended to determine those with different  $\Lambda$ .
- `[effective theory]` Is a logical variable which controls whether or not the effective field theory is used in the calculation of the DM process. If this value is set to `.false.` then one must specify the mass of the light mediator and its width (see below for more details).
- `[Yukawa Scalar couplings]` Is a logical variable which determines if the scalar and pseudo scalar operators scale with the factor  $m_q/\Lambda$  ( `.true.` ) or 1 ( `.false.` ).

- `[Left handed DM couplings]` and `[Right handed DM couplings]` These variables determine the coupling of the various flavours of quarks to the DM operators. The default value is 1. Note that the code uses the fact that vector operators scale as  $(L + R)$  and axial operators scale as  $(L - R)$  in constructing cross sections. Therefore you should be careful if modifying these parameters. For the axial and pseudo scalar operators the code will set the right-handed couplings to be the negative of the left handed input couplings (if this is not already the case from the setup) and inform the user it has done so. The most likely reason to want to change these values is to inspect individual flavour operators separately, i.e. to investigate an operator which only couples to up quarks one would set all couplings to 0d0 apart from `[up type]` which would be left as 1d0.
- `[mediator mass]` If `[effective theory]` is set to `.false.` this variable controls the mass of the mediating particle.
- `[mediator width]` If `[effective theory]` is set to `.false.` this variable controls the width of the mediating particle
- `[g_x]` If `[effective theory]` is set to `.false.` this variable controls the coupling of the mediating particle to the DM.
- `[g_q]` If `[effective theory]` is set to `.false.` this variable controls the coupling of the mediating particle to the quarks.

We now discuss some details of the specific DM process.

- Processes 800 and 820 produce the mono-jet or mono-photon signature through the following vector operator,

$$\mathcal{O}_V = \frac{(\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q)}{\Lambda^2}, \quad (16)$$

These processes are available at NLO and include the usual treatment of photons. See for instance the  $V\gamma$  processes ( $\sim 300$ ) in this manual for more details on photon setup in MCFM. As discussed above the code will calculate left and right-handed helicity amplitudes and build the vector operators from  $(L + R)$ . Therefore you should ensure that the Left and right-handed couplings are equal in `dm_parameters.DAT`. Processes 840 and 845 represent the production of DM plus two jets or DM plus one jet and one photon and are available at LO.

- Processes 801 and 821 produce the mono-jet or mono-photon signature through the following axial-vector operator,

$$\mathcal{O}_A = \frac{(\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5q)}{\Lambda^2}, \quad (17)$$

These processes are available at NLO and include the usual treatment of photons. See for instance the  $V\gamma$  processes ( $\sim 300$ ) in this manual for more details on photon setup in MCFM. As discussed above the code will calculate left and right-handed helicity amplitudes and build the axial vector operators from  $(L - R)$ . By default the code will enforce the right handed couplings to equal to the negative of the left handed couplings, if this is not already the case in `dm_parameters.DAT`. Therefore the user does not have to change this file when switching between vector and axial vector operators. Processes 841 and 846 represent the production of DM plus two jets or DM plus one jet and one photon and are available at LO.

- Processes 802 and 822 produce the mono-jet or mono-photon signature through the following scalar operator,

$$\mathcal{O}_S = \frac{\Delta(\bar{\chi}\chi)(\bar{q}q)}{\Lambda^2}, \quad (18)$$

These processes are available at NLO and include the usual treatment of photons. See for instance the  $V\gamma$  processes ( $\sim 300$ ) in this manual for more details on photon setup in MCFM. As discussed above the code will calculate left and right-handed helicity amplitudes and build the vector operators from  $(L + R)$ . Therefore you should ensure that the Left and right-handed couplings are equal in `dm_parameters.DAT`. For these processes  $\Delta$  is fixed from the value of `[Yukawa Scalar Couplings]` if this is `.true.` then  $\Delta = m_q/\Lambda$  else  $\Delta = 1$ .

Processes 842 and 847 represent the production of DM plus two jets or DM plus one jet and one photon and are available at LO.

- Processes 803 and 823 produce the mono-jet or mono-photon signature through the following pseudo-scalar operator,

$$\mathcal{O}_{PS} = \frac{m_q(\bar{\chi}\gamma_5\chi)(\bar{q}\gamma_5q)}{\Lambda^3}. \quad (19)$$

These processes are available at NLO and include the usual treatment of photons. See for instance the  $V\gamma$  processes ( $\sim 300$ ) in this manual for more details on photon setup in MCFM. As discussed above the code will calculate left and right-handed helicity amplitudes and build the pseudo scalar operators from  $(L - R)$ . By default the code will enforce the right handed couplings to equal to the negative of the left handed couplings, if this is not already the case in `dm_parameters.DAT`. Therefore the user does not have to change this file when switching between scalar and pseudo scalar operators. Processes 841 and 846 represent the production of DM plus two jets or DM plus one jet and one photon and are available at LO. For these processes  $\Delta$  is fixed from the value of `[Yukawa Scalar Couplings]` if this is `.true.` then  $\Delta = m_q/\Lambda$  else  $\Delta = 1$ .

Processes 843 and 848 represent the production of DM plus two jets or DM plus one jet and one photon and are available at LO.

- Process 804 produces the mono-jet signature through the following gluon induced operator,

$$\mathcal{O}_g = \alpha_s \frac{(\chi\bar{\chi})(G_a^{\mu\nu}G_{a,\mu\nu})}{\Lambda^3}, \quad (20)$$

This process is available at NLO. Process 844 represents the production of DM plus two jets and is available at LO. Since this operator is higher dimensional, extensions to a theory in which there is a light mediator requires the definition of two new scales (one for the EFT in the loop defining the operator). In this version we therefore do not consider in a theory with a light mediator.

- Process 805 is a separate case of the scalar operator for top quarks

$$\mathcal{O}_S^{m_t} = \frac{m_t(\bar{\chi}\chi)(\bar{q}q)}{\Lambda^3}, \quad (21)$$

This process is available at LO and proceeds through a gluon loop.

## A. List of historical PDF sets

The availability of a number of historical PDF sets is retained in the code. These should typically not be used in modern analyses, but they may be helpful for comparison with older codes or in specialized cases.

These distributions, together with their associated  $\alpha_S(M_Z)$  values, are given in Tables 30 and 31. For the older distributions, where the coupling was specified by  $\Lambda$  this requires some calculation and/or guesswork.

mrstqed	0.1205	hep-ph/0411040	mrs02n1	0.1197	hep-ph/0211080
mrs02nn	0.1154	hep-ph/0211080	mrs4nf3	0.1083	hep-ph/0603143
mrs4lf3	0.1186	hep-ph/0603143	mrs4nf4	0.1153	hep-ph/0603143
mrs4lf4	0.1251	hep-ph/0603143	mrs0119	0.119	hep-ph/0110215
mrs0117	0.117	hep-ph/0110215	mrs0121	0.121	hep-ph/0110215
mrs01_j	0.121	hep-ph/0110215	mrs011o	0.130	hep-ph/0201127
mrs99_1	0.1175	hep-ph/9907231	mrs99_2	0.1175	hep-ph/9907231
mrs99_3	0.1175	hep-ph/9907231	mrs99_4	0.1125	hep-ph/9907231
mrs99_5	0.1225	hep-ph/9907231	mrs99_6	0.1178	hep-ph/9907231
mrs99_7	0.1171	hep-ph/9907231	mrs99_8	0.1175	hep-ph/9907231
mrs99_9	0.1175	hep-ph/9907231	mrs9910	0.1175	hep-ph/9907231
mrs9911	0.1175	hep-ph/9907231	mrs9912	0.1175	hep-ph/9907231
mrs98z1	0.1175	hep-ph/9803445	mrs98z2	0.1175	hep-ph/9803445
mrs98z3	0.1175	hep-ph/9803445	mrs98z4	0.1125	hep-ph/9803445
mtungb1	0.109	hep-ph/9803445	mrs98z5	0.1225	hep-ph/9803445
mrs96r1	0.113	PLB387 (1996) 419	mrs96r2	0.120	PLB387 (1996) 419
mrs96r3	0.113	PLB387 (1996) 419	mrs96r4	0.120	PLB387 (1996) 419
mrs95ap	0.1127	PLB354 (1995) 155	mrs95_g	0.1148	PLB354 (1995) 155
hmrs90e	0.09838	Durham DTP-90-04	hmrs90b	0.10796	Durham DTP-90-04

Table 30: Historical MRS-type pdf sets, their corresponding values of  $\alpha_S(M_Z)$  and a reference to the paper or preprint that describes their origin.

## B. MCFM references

As general references for NLO computations with MCFM, please use:

- J. M. Campbell and R. K. Ellis,  
*“An update on vector boson pair production at hadron colliders,”*  
 Phys. Rev. D **60**, 113006 (1999) [arXiv:hep-ph/9905386].
- J. M. Campbell, R. K. Ellis and C. Williams,  
*“Vector boson pair production at the LHC,”*  
 JHEP **1107**, 018 (2011) [arXiv:1105.0020 [hep-ph]].

cteq61m	0.118	hep-ph/0303013	cteq66m	0.118	0802.0007 [hep-ph]
cteq6_d	0.118	hep-ph/0201195	cteq6_m	0.118	hep-ph/0201195
cteq6l1	0.130	hep-ph/0201195	cteq6_l	0.118	hep-ph/0201195
cteq5f3	0.106	hep-ph/9903282	cteq5hq	0.118	hep-ph/9903282
cteq5_m	0.118	hep-ph/9903282	cteq5f4	0.112	hep-ph/9903282
cteq5_l	0.127	hep-ph/9903282	cteq5_d	0.118	hep-ph/9903282
cteq5hj	0.118	hep-ph/9903282	cteq5l1	0.127	hep-ph/9903282
ctq5hq1	0.118	hep-ph/9903282	cteq5m1	0.118	hep-ph/9903282
cteq4hj	0.116	hep-ph/9606399	cteq4a5	0.122	hep-ph/9606399
cteq4_m	0.116	hep-ph/9606399	cteq4lq	0.114	hep-ph/9606399
cteq4_l	0.132	hep-ph/9606399	cteq4_d	0.116	hep-ph/9606399
cteq4a2	0.113	hep-ph/9606399	cteq4a1	0.110	hep-ph/9606399
cteq4a4	0.119	hep-ph/9606399	cteq4a3	0.116	hep-ph/9606399
cteq3_l	0.112	MSU-HEP/41024	cteq3_m	0.112	MSU-HEP/41024
			cteq3_d	0.112	MSU-HEP/41024

Table 31: Historical CTEQ-type pdf sets, their corresponding values of  $\alpha_S(M_Z)$  and a reference to the paper or preprint that describes their origin.

- J. M. Campbell, R. K. Ellis and W. Giele,  
“A *Multi-Threaded Version of MCFM*”,  
EPJ **C75**, 246 (2015) [arXiv:1503.06182 [hep-ph]].

When using MCFM 8.0, or later versions, for NNLO calculations of color-singlet processes please refer to:

- R. Boughezal, J. M. Campbell, R. K. Ellis,  
C. Focke, W. Giele, X. Liu, F. Petriello and C. Williams,  
“*Color singlet production at NNLO in MCFM*”, arXiv:1605.08011.

For calculations of electroweak corrections please refer to:

- J. M. Campbell, D. Wackerroth and J. Zhou,  
“*A Study of Weak Corrections to Drell-Yan, Top-quark pair and Di-jet Production at High Energies with MCFM*”, arXiv:1608.03356.

When using MCFM 9.0, or later versions please also refer to:

- John Campbell and Tobias Neumann. “Precision phenomenology with MCFM” (2019).  
arXiv: 1909.09117 [hep-ph]

Additional references to the program may be used depending on the process under study. The relevant papers are:

- J. M. Campbell and R. K. Ellis,  
“*Radiative corrections to Z b anti-b production,*”  
Phys. Rev. D **62**, 114012 (2000) [arXiv:hep-ph/0006304].
- J. Campbell and R. K. Ellis,  
“*Next-to-leading order corrections to W + 2jet and Z + 2jet production at hadron*

- colliders,*"  
 Phys. Rev. D **65**, 113007 (2002) [arXiv:hep-ph/0202176].
- J. Campbell, R. K. Ellis, F. Maltoni and S. Willenbrock,  
 "Higgs boson production in association with a single bottom quark,"  
 Phys. Rev. D **67**, 095002 (2003) [arXiv:hep-ph/0204093].
  - J. Campbell, R. K. Ellis and D. L. Rainwater,  
 "Next-to-leading order QCD predictions for  $W + 2jet$  and  $Z + 2jet$  production at the  
 CERN LHC,"  
 Phys. Rev. D **68**, 094021 (2003) [arXiv:hep-ph/0308195].
  - J. Campbell, R. K. Ellis, F. Maltoni and S. Willenbrock,  
 "Associated production of a Z boson and a single heavy-quark jet,"  
 Phys. Rev. D **69**, 074021 (2004) [arXiv:hep-ph/0312024].
  - E. L. Berger and J. Campbell,  
 "Higgs boson production in weak boson fusion at next-to-leading order,"  
 Phys. Rev. D **70**, 073011 (2004) [arXiv:hep-ph/0403194].
  - J. Campbell, R. K. Ellis and F. Tramontano,  
 "Single top production and decay at next-to-leading order,"  
 Phys. Rev. D **70**, 094012 (2004) [arXiv:hep-ph/0408158].
  - J. Campbell and F. Tramontano,  
 "Next-to-leading order corrections to  $Wt$  production and decay,"  
 Nucl. Phys. B **726**, 109 (2005) [arXiv:hep-ph/0506289].
  - J. Campbell, R.K. Ellis, F. Maltoni and S. Willenbrock,  
 "Production of a Z boson and two jets with one heavy quark tag,"  
 Phys. Rev. D **73**, 054007 (2006) [arXiv:hep-ph/0510362].
  - J. M. Campbell, R. Frederix, F. Maltoni and F. Tramontano,  
 "t-channel single top production at hadron colliders,"  
 Phys. Rev. Lett. **102** (2009) 182003, [arXiv:0903.0005 [hep-ph]].
  - J. M. Campbell, R. K. Ellis and G. Zanderighi,  
 "Next-to-leading order Higgs + 2 jet production via gluon fusion,"  
 JHEP **0610**, 028 (2006) [arXiv:hep-ph/0608194].
  - J. Campbell, R.K. Ellis, F. Maltoni and S. Willenbrock,  
 "Production of a W boson and two jets with one b-quark tag,"  
 Phys. Rev. D **75**, 054015 (2007) [arXiv:hep-ph/0611348].
  - J. M. Campbell, R. K. Ellis, F. Febres Cordero, F. Maltoni, L. Reina, D. Wackerroth  
 and S. Willenbrock,  
 "Associated Production of a W Boson and One b Jet,"  
 Phys. Rev. D **79**, 034023 (2009) [arXiv:0809.3003 [hep-ph]].
  - J. M. Campbell, R. K. Ellis and C. Williams,  
 "Hadronic production of a Higgs boson and two jets at next-to-leading order,"  
 Phys. Rev. D **81** 074023 (2010), [arXiv:1001.4495 [hep-ph]].



- S. Badger, J. M. Campbell and R. K. Ellis,  
“QCD corrections to the hadronic production of a heavy quark pair and a W-boson including decay correlations,”  
JHEP **1103**, 027 (2011) [arXiv:1011.6647 [hep-ph]].
- F. Caola, J. M. Campbell, F. Febres Cordero, L. Reina and D. Wackerroth,  
“NLO QCD predictions for  $W + 1$  jet and  $W + 2$  jet production with at least one b jet at the 7 TeV LHC,”  
arXiv:1107.3714 [hep-ph].
- J. M. Campbell, R. K. Ellis and C. Williams,  
“Gluon-gluon contributions to  $W^+ W^-$  production and Higgs interference effects,”  
JHEP **1110**, 005 (2011), arXiv:1107.5569 [hep-ph].
- J. M. Campbell and R. K. Ellis,  
“Top-quark processes at NLO in production and decay,”  
arXiv:1204.1513 [hep-ph], FERMILAB-PUB-12-078-T.
- J. M. Campbell and R. K. Ellis,  
“ $t\bar{t}W^\pm$  production and decay at NLO,”  
JHEP **1207**, 052 (2012), [arXiv:1204.5678 [hep-ph]].
- J. M. Campbell, H. B. Hartanto and C. Williams  
“Next-to-leading order predictions for  $Z\gamma$ +jet and  $Z\gamma\gamma$  final states at the LHC” JHEP **1211**, 162 (2012), [arXiv:1208.0566 [hep-ph]].
- P. J. Fox and C. Williams  
“Next-to-leading order predictions for Dark Matter Production at Hadron Colliders”  
[arXiv:1211.6390 [hep-ph]].
- J. M. Campbell, R. K. Ellis and R. Röntsch, “Single top production in association with a Z boson at the LHC,” arXiv:1302.3856 [hep-ph].
- J. M. Campbell, R. K. Ellis and C. Williams, “Associated Production of a Higgs Boson at NNLO”, arXiv:1601.00658 [hep-ph].
- J. M. Campbell, R. K. Ellis, Ye Li and C. Williams, “Predictions for Diphoton Production at the LHC through NNLO in QCD”, arXiv:1603.02663 [hep-ph].
- Tobias Neumann and Ciaran Williams. “The Higgs boson at high  $p_T$ ”. *Phys. Rev. D* **95**.1 (2017), p. 014004. DOI: 10.1103/PhysRevD.95.014004. arXiv: 1609.00367 [hep-ph]
- Tobias Neumann. “NLO Higgs+jet production at large transverse momenta including top quark mass effects”. *J. Phys. Comm.* **2**.9 (2018), p. 095017. DOI: 10.1088/2399-6528/aadfbf. arXiv: 1802.02981 [hep-ph]
- John M. Campbell, Tobias Neumann, and Ciaran Williams. “ $Z\gamma$  production at NNLO including anomalous couplings”. *Submitted to: JHEP* (2017). arXiv: 1708.02925 [hep-ph]
- Tobias Neumann and Zack Edward Sullivan. “Off-Shell Single-Top-Quark Production in the Standard Model Effective Field Theory”. *JHEP* **06** (2019), p. 022. DOI: 10.1007/JHEP06(2019)022. arXiv: 1903.11023 [hep-ph]

## C. Processes included in MCFM

nproc	$f(p_1) + f(p_2) \rightarrow \dots$	Order
1	$W^+(\rightarrow \nu(p_3) + e^+(p_4))$	NNLO
6	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4))$	NNLO
11	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + f(p_5)$	NLO
12	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{b}(p_5)$	NLO
13	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{c}(p_5)$	NLO
14	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{c}(p_5)$ [massless]	LO
16	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + f(p_5)$	NLO
17	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5)$	NLO
18	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + c(p_5)$	NLO
19	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + c(p_5)$ [massless]	LO
20	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6)$ [massive]	NLO
21	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6)$	NLO
22	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + f(p_5) + f(p_6)$	NLO
23	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + f(p_5) + f(p_6) + f(p_7)$	LO
24	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6) + f(p_7)$	LO
25	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + \bar{b}(p_6)$ [massive]	NLO
26	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + \bar{b}(p_6)$	NLO
27	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + f(p_5) + f(p_6)$	NLO
28	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + f(p_5) + f(p_6) + f(p_7)$	LO
29	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + \bar{b}(p_6) + f(p_7)$	LO
31	$Z(\rightarrow e^-(p_3) + e^+(p_4))$	NNLO
310	$Z(\rightarrow e^-(p_3) + e^+(p_4))$ [photon induced]	LO
32	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4)))$	NNLO
33	$Z(\rightarrow b(p_3) + \bar{b}(p_4))$	NLO
34	$Z(\rightarrow 3 \times (d(p_5) + \bar{d}(p_6)))$	NLO
35	$Z(\rightarrow 2 \times (u(p_5) + \bar{u}(p_6)))$	NLO
36	$Z \rightarrow t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{b}(p_6) + e^-(p_7) + \bar{\nu}(p_8))$	LO
41	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + f(p_5)$	NLO
42	$Z_0(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + f(p_5)$	NLO
43	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5)$	NLO
44	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + f(p_5) + f(p_6)$	NLO
45	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + f(p_5) + f(p_6) + f(p_7)$	LO
46	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4)) + f(p_5) + f(p_6)$	NLO
47	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4)) + f(p_5) + f(p_6) + f(p_7)$	LO
50	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \bar{b}(p_5) + b(p_6)$ [massive]	LO
51	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6)$	NLO
52	$Z_0(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + b(p_5) + \bar{b}(p_6)$	NLO
53	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + b(p_5) + \bar{b}(p_6)$	NLO
54	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6) + f(p_7)$	LO

## References

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56	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5) + \bar{c}(p_6)$	NLO
61	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^-(\rightarrow e^-(p_5) + \bar{\nu}(p_6))$	NLO
62	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^-(\rightarrow q(p_5) + \bar{q}(p_6))$	NLO
63	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^-(\rightarrow q(p_5) + \bar{q}(p_6))[\text{rad.in.dk}]$	NLO
64	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4))W^+(\rightarrow q(p_5) + \bar{q}(p_6))$	NLO
65	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4))W^+(\rightarrow q(p_5) + \bar{q}(p_6))[\text{rad.in.dk}]$	NLO
66	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^-(\rightarrow e^-(p_5) + \bar{\nu}(p_6)) + f(p_7)$	LO
69	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^-(\rightarrow e^-(p_5) + \bar{\nu}(p_6))[\text{no pol}]$	LO
71	$W^+(\rightarrow \nu(p_3) + \mu^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6))$	NLO
72	$W^+(\rightarrow \nu(p_3) + \mu^+(p_4)) + Z(\rightarrow 3 \times (\nu_e(p_5) + \bar{\nu}_e(p_6)))$	NLO
73	$W^+(\rightarrow \nu(p_3) + \mu^+(p_4)) + Z(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
74	$W^+(\rightarrow \nu(p_3) + \mu^+(p_4)) + Z(\rightarrow 3 \times (d(p_5) + \bar{d}(p_6)))$	NLO
75	$W^+(\rightarrow \nu(p_3) + \mu^+(p_4)) + Z(\rightarrow 2 \times (u(p_5) + \bar{u}(p_6)))$	NLO
76	$W^-(\rightarrow \mu^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6))$	NLO
77	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow 3 \times (\nu_e(p_5) + \bar{\nu}_e(p_6)))$	NLO
78	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
79	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow 3 \times (d(p_5) + \bar{d}(p_6)))$	NLO
80	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow 2 \times (u(p_5) + \bar{u}(p_6)))$	NLO
81	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6))$	NLO
82	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow 3 \times (\nu(p_5) + \bar{\nu}(p_6)))$	NLO
83	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
84	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + Z(\rightarrow 3 \times (\nu(p_5) + \bar{\nu}(p_6)))$	NLO
85	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow 3 \times (\nu(p_5) + \bar{\nu}(p_6))) + f(p_7)$	LO
86	$Z(\rightarrow \mu^-(p_3) + \mu^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6))[\text{no gamma}^*]$	NLO
87	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow 3 \times (\nu(p_5) + \bar{\nu}(p_6)))[\text{no gamma}^*]$	NLO
88	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow b(p_5) + \bar{b}(p_6))[\text{no gamma}^*]$	NLO
89	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + Z(\rightarrow 3 \times (\nu(p_5) + \bar{\nu}(p_6)))[\text{no gamma}^*]$	NLO
90	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6))$	NLO
91	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow \tau(p_5) + \bar{\tau}(p_6))$	NNLO
92	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NNLO
920	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6)) [\text{rad.in.dk}]$	NLO
93	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow \gamma(p_5) + \gamma(p_6))$	NNLO
94	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow W^+(\nu(p_5), e^+(p_6))W^-(e^-(p_7), \bar{\nu}(p_8)))$	NNLO
95	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + H(\rightarrow Z(e^-(p_5), e^+(p_6)) + Z(\mu^-(p_7), \mu^+(p_8)))$	NNLO
96	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow \tau(p_5) + \bar{\tau}(p_6))$	NNLO
97	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NNLO
970	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))[\text{rad.in.dk}]$	NLO
98	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow \gamma(p_5) + \gamma(p_6))$	NNLO
99	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow W^+(\nu(p_5), e^+(p_6))W^-(e^-(p_7), \bar{\nu}(p_8)))$	NNLO
100	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow Z(e^-(p_5), e^+(p_6)) + Z(\mu^-(p_7), \mu^+(p_8)))$	NNLO

101	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NNLO
102	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
103	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
104	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + H(\rightarrow \gamma(p_5) + \gamma(p_6))$	NNLO
105	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + H(\rightarrow \gamma(p_5) + \gamma(p_6))$	NLO
106	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + H(\rightarrow W^+(\nu(p_5), e^+(p_6))W^-(e^-(p_7), \bar{\nu}(p_8)))$	NNLO
107	$Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + H(\rightarrow W^+(\nu(p_5), e^+(p_6))W^-(e^-(p_7), \bar{\nu}(p_8)))$	NLO
108	$Z(\rightarrow b(p_3) + \bar{b}(p_4)) + H(\rightarrow W^+(\nu(p_5), e^+(p_6))W^-(e^-(p_7), \bar{\nu}(p_8)))$	NLO
109	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + H(\rightarrow Z(e^-(p_5), e^+(p_6)) + Z(\rightarrow \mu^-(p_7), \mu^+(p_8)))$	NLO
110	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + H(\rightarrow \tau^-(p_5)\tau^+(p_6))$	NNLO
111	$H(\rightarrow b(p_3) + \bar{b}(p_4))$	NNLO
112	$H(\rightarrow \tau^-(p_3) + \tau^+(p_4))$	NNLO
113	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6)))$	NLO
114	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(q(p_5) + \bar{q}(p_6)))$	NLO
115	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(q(p_5) + \bar{q}(p_6)))[rad.in.dk]$	NLO
116	$H(\rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6)))$	NLO
117	$H(\rightarrow Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6)))$	NLO
118	$H(\rightarrow Z(\rightarrow \mu^-(p_3) + \mu^+(p_4)) + Z(\rightarrow b(p_5) + \bar{b}(p_6)))$	NLO
119	$H(\rightarrow \gamma(p_3) + \gamma(p_4))$	NNLO
120	$H(\rightarrow Z(\rightarrow \mu^-(p_3) + \mu^+(p_4)) + \gamma(p_5))$	NLO
121	$H(\rightarrow Z(\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4)))) + \gamma(p_5)$	NLO
123	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6)))$ [t, b loops, exact]	LO
124	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6)))$ [only H, gg→WW int]	LO
125	$H(\rightarrow W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6)))$ [ $ H ^2$ and H,gg→WW int]	LO
126	$W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6))$ [gg only, (H + gg→WW) squared]	LO
128	$H(\rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6)))$ [t, b loops, exact]	LO
129	$H(\rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6)))$ [only H, gg→ZZ int]	LO
130	$H(\rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6)))$ [ $ H ^2$ and H,gg→ZZ int]	LO
131	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6))$ [gg only, $ H + gg \rightarrow ZZ ^2$ ]	LO
132	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6))$ [(gg→ZZ) squared]	LO
1281	$H(\rightarrow e^-(p_3) + e^+(p_4)\nu_e(p_5) + \bar{\nu}_e(p_6))$ [top, bottom loops, exact]	LO
1311	$e^-(p_3) + e^+(p_4) + \nu_e(p_5) + \bar{\nu}_e(p_6)$ [gg only, (H + gg→ZZ) squared]	LO
1321	$e^-(p_3) + e^+(p_4) + \nu_e(p_5) + \bar{\nu}_e(p_6)$ [(gg→ZZ) squared]	LO
1282	$H(\rightarrow e^-(p_3) + e^+(p_4) + \nu(p_5) + \bar{\nu}(p_6))$ [top, bottom loops, exact]	LO
1312	$e^-(p_3) + e^+(p_4) + \nu(p_5) + \bar{\nu}(p_6)$ [gg only, (H + gg→ZZ) squared]	LO
1322	$e^-(p_3) + e^+(p_4) + \nu(p_5) + \bar{\nu}(p_6)$ [(gg→ZZ) squared]	LO
133	$H(\rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + Z(\rightarrow \mu^-(p_5) + \mu^+(p_6) + f(p_7)))$ [intf,no $p_7$ cut]	LO

136	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + b(p_5)(+g(p_6))$	NLO
137	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + \bar{b}(p_5)(+b(p_6))$	(REAL)
138	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + b(p_5) + \bar{b}(p_6)$ [both observed]	(REAL)
141	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$	NLO
142	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$ [rad.in.dk]	NLO
143	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8)) + f(p_9)$	LO
144	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$ (uncorr)	NLO
145	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$ [rad.in.dk],uncorr	NLO
146	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + q(p_7) + \bar{q}(p_8))$	NLO
147	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + q(p_7) + \bar{q}(p_8))$ [rad.in.top.dk]	NLO
148	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + q(p_7) + \bar{q}(p_8))$ [rad.in.W.dk]	NLO
149	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$	NLO
150	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$ [rad.in.top.dk]	NLO
151	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8))$ [rad.in.W.dk]	NLO
157	$t\bar{t}$ [for total Xsect]	NLO
158	$b\bar{b}$ [for total Xsect]	NLO
159	$c\bar{c}$ [for total Xsect]	NLO
160	$t\bar{t} + g$ [for total Xsect]	LO
161	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + q(p_6)$ [t-channel]	NLO
162	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + q(p_6)$ [decay]	NLO
163	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + q(p_6)$ [t-channel] $mb > 0$	NLO
164	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + q(p_6)$ [full off-shell t-channel] $mb = 0$	NLO
166	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + q(p_6)$ [t-channel]	NLO
167	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + q(p_6)$ [rad.in.dk]	NLO
168	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + q(p_6)$ [t-channel] $mb > 0$	NLO
169	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + q(p_6)$ [full off-shell t-channel] $mb = 0$	NLO
171	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{b}(p_6)$ [s-channel]	NLO
172	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{b}(p_6)$ [decay]	NLO
176	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + b(p_6)$ [s-channel]	NLO
177	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + b(p_6)$ [rad.in.dk]	NLO
180	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(p_5)$	NLO
181	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(\nu(p_5) + e^+(p_6) + b(p_7))$	NLO
182	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(\nu(p_5) + e^+(p_6) + b(p_7))$ [rad.in.dk]	NLO
183	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(\nu(p_5) + e^+(p_6) + b(p_7)) + b(p_8)$	LO
184	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(p_5) + b(p_6)$ [massive b]	LO
185	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{t}(p_5)$	NLO
186	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{t}(e^-(p_5) + \bar{\nu}(p_6) + \bar{b}(p_7))$	NLO
187	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \bar{t}(e^-(p_5) + \bar{\nu}(p_6) + \bar{b}(p_7))$ [rad.in.dk]	NLO

190	$f(p_3) + f(p_4)$	LO
191	$f(p_3) + f(p_4)$ [mixed QCD/EW]	NLO
200	$H + f(p_5)$ , see section 6.39	NLO
201	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5)$ [full mt dep.]	LO
202	$H(\rightarrow \tau^-(p_3) + \tau^+(p_4)) + f(p_5)$ [full mt dep.]	LO
203	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5)$	NNLO
204	$H(\rightarrow \tau^-(p_3) + \tau^+(p_4)) + f(p_5)$	NNLO
205	$H(\rightarrow Z(\mu^-(p_3) + \mu^+(p_4)) + \gamma(p_5)) + f(p_5)$	NLO
206	$A(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5)$ [full mt dep.]	LO
207	$A(\rightarrow \tau^-(p_3) + \tau^+(p_4)) + f(p_5)$ [full mt dep.]	LO
208	$H(\rightarrow W^+(\nu(p_3), e^+(p_4))W^-(e^-(p_5), \bar{\nu}(p_6))) + f(p_7)$	NLO
209	$H(\rightarrow Z(\rightarrow e^-(p_3), e^+(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7)$	NLO
210	$H(\rightarrow \gamma(p_3) + \gamma(p_4)) + f(p_5)$	NNLO
211	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5) + f(p_6)$ [WBF]	NLO
212	$H(\rightarrow \tau^-(p_3) + \tau^+(p_4)) + f(p_5) + f(p_6)$ [WBF]	NLO
213	$H(\rightarrow W^+(\nu(p_3), e^+(p_4))W^-(e^-(p_5), \bar{\nu}(p_6))) + f(p_7) + f(p_8)$ [WBF]	NLO
214	$H(\rightarrow Z(\rightarrow e^-(p_3), e^+(p_4)) + Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [WBF]	NLO
215	$H(\rightarrow \gamma(p_3) + \gamma(p_4)) + f(p_5) + f(p_6)$ [WBF]	NLO
216	$H(\rightarrow b(p_3) + \bar{b}(p_4)) + f(p_5) + f(p_6) + f(p_7)$ [WBF+jet]	LO
217	$H(\rightarrow \tau^-(p_3) + \tau^+(p_4)) + f(p_5) + f(p_6) + f(p_7)$ [WBF+jet]	LO
221	$\tau^-(\rightarrow e^-(p_3) + \bar{\nu}_e(p_4) + \nu_\tau(p_5)) + \tau^+(\rightarrow \bar{\nu}_\tau(p_6) + \nu_e(p_7) + e^+(p_8))$	LO
220	$Z(\rightarrow e^-(p_3), e^+(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [weak]'	LO
2201	$Z(\rightarrow e^-(p_3), e^+(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [strong]'	LO
222	$Z(\rightarrow e^-(p_3), e^+(p_4))Z(\rightarrow \nu_\mu(p_5), \bar{\nu}_\mu(p_6))) + f(p_7) + f(p_8)$ [weak]'	LO
2221	$Z(\rightarrow e^-(p_3), e^+(p_4))Z(\rightarrow \nu_\mu(p_5), \bar{\nu}_\mu(p_6))) + f(p_7) + f(p_8)$ [strong]'	LO
224	$W^-(e^-(p_3), \bar{\nu}_e(p_6)))W^+(\nu_\mu(p_5), \mu^+(p_4)) + f(p_7) + f(p_8)$ [weak]'	LO
2241	$W^-(e^-(p_3), \bar{\nu}_e(p_6)))W^+(\nu_\mu(p_5), \mu^+(p_4)) + f(p_7) + f(p_8)$ [strong]'	LO
226	$e^-(p_3) + e^+(p_4) + \nu_e(p_5) + \bar{\nu}_e(p_6) + f(p_7) + f(p_8)$ [weak]'	LO
228	$W^+(\nu_e(p_3), e^+(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)$ [weak]'	LO
2281	$W^+(\nu_e(p_3), e^+(p_4))W^+(\nu_\mu(p_5), \mu^+(p_6)) + f(p_7) + f(p_8)$ [strong]'	LO
229	$W^-(e^-(p_3), \bar{\nu}_e(p_4))W^-(\mu^-(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)$ [weak]'	LO
2291	$W^-(e^-(p_3), \bar{\nu}_e(p_4))W^-(\mu^-(p_5), \bar{\nu}_\mu(p_6)) + f(p_7) + f(p_8)$ [strong]'	LO
223	$W^+(\nu_e(p_3), e^+(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [weak]'	LO
2231	$W^+(\nu_e(p_3), e^+(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [strong]'	LO
225	$W^-(e^-(p_3), \bar{\nu}_e(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [weak]'	LO
2251	$W^-(e^-(p_3), \bar{\nu}_e(p_4))Z(\rightarrow \mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$ [strong]'	LO



231	$t(p_3) + \bar{b}(p_4) + q(p_5)$ [t-channel]	NLO
232	$t(p_3) + \bar{b}(p_4) + q(p_5) + q(p_6)$ [t-channel]	LO
233	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{b}(p_6) + q(p_7)$ [t-channel]	NLO
234	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{b}(p_6) + q(p_7)$ [t-channel, rad.in.dk]	NLO
235	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{b}(p_6) + q(p_7) + f(p_8)$ [t-channel]	LO
236	$\bar{t}(p_3) + b(p_4) + q(p_5)$ [t-channel]	NLO
237	$\bar{t}(p_3) + b(p_4) + q(p_5) + q(p_6)$ [t-channel]	LO
238	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + b(p_6) + q(p_7)$ [t-channel]	NLO
239	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + b(p_6) + q(p_7)$ [t-channel, rad.in.dk]	NLO
240	$\bar{t}(\rightarrow e^-(p_3) + \bar{\nu}(p_4) + \bar{b}(p_5)) + b(p_6) + q(p_7) + f(p_8)$ [t-channel]	LO
251	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^+(\rightarrow \nu(p_5) + e^+(p_6)) + f(p_7) + f(p_8)$	LO
252	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + W^+(\rightarrow \nu(p_5) + e^+(p_6)) + f(p_7) + f(p_8) + f(p_9)$	LO
253	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + f(p_7) + f(p_8)$	LO
254	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + f(p_7) + f(p_8)$	LO
255	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + b(p_7) + f(p_8)$	LO
256	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + b(p_7) + f(p_8)$	LO
259	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + b(p_7) + b(p_8)$	LO
260	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + Z(\rightarrow e^-(p_5) + e^+(p_6)) + b(p_7) + b(p_8)$	LO
261	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5)$	NLO
262	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5)$	NLO
263	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \bar{b}(p_5) + b(p_6)$ [1 b-tag]	LO
264	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \bar{c}(p_5) + c(p_6)$ [1 c-tag]	LO
266	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5)(+\bar{b}(p_6))$	NLO
267	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5)(+\bar{c}(p_6))$	NLO

270	$H(\gamma(p_3) + \gamma(p_4)) + f(p_5) + f(p_6)$ [in heavy top limit]	NLO
271	$H(b(p_3) + \bar{b}(p_4)) + f(p_5) + f(p_6)$ [in heavy top limit]	NLO
272	$H(\tau^-(p_3) + \tau^+(p_4)) + f(p_5) + f(p_6)$ [in heavy top limit]	NLO
273	$H(\rightarrow W^+(\nu(p_3), e^+(p_4))W^-(e^-(p_5), \bar{\nu}(p_6))) + f(p_7) + f(p_8)$	NLO
274	$H(\rightarrow Z(e^-(p_3), e^+(p_4))Z(\mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8)$	NLO
275	$H(b(p_3) + \bar{b}(p_4)) + f(p_5) + f(p_6) + f(p_7)$ [in heavy top limit]	LO
276	$H(\tau^-(p_3) + \tau^+(p_4)) + f(p_5) + f(p_6) + f(p_7)$ [in heavy top limit]	LO
278	$H(\rightarrow W^+(\nu(p_3), e^+(p_4))W^-(e^-(p_5), \bar{\nu}(p_6))) + f(p_7) + f(p_8) + f(p_9)$	LO
279	$H(\rightarrow Z(e^-(p_3), e^+(p_4))Z(\mu^-(p_5), \mu^+(p_6))) + f(p_7) + f(p_8) + f(p_9)$	LO
280	$\gamma(p_3) + f(p_4)$	NLO+F
282	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + f(p_4) + f(p_5)$	LO
283	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + b(p_4)$	LO
284	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + c(p_4)$	LO
285	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + \gamma(p_4)$	NLO+F, NNLO
286	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + \gamma(p_4) + f(p_5)$	LO
287	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + \gamma(p_4) + \gamma(p_5)$	NLO+F
289	$f(p_1) + f(p_2) \rightarrow \gamma(p_3) + \gamma(p_4) + \gamma(p_5) + \gamma(p_6)$	NLO+F
290	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \gamma(p_5)$	NLO+F
292	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \gamma(p_5) + f(p_6)$	LO
295	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + \gamma(p_5)$	NLO+F
297	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + \gamma(p_5) + f(p_6)$	LO
300	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \gamma(p_5)$	NNLO
301	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \gamma(p_5) + \gamma(p_6)$	NLO + F
302	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \gamma(p_5) + f(p_6)$	NLO
303	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \gamma(p_5) + \gamma(p_6) + f(p_7)$	LO
304	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + \gamma(p_5) + f(p_6) + f(p_7)$	LO
305	$Z(\rightarrow 3(\nu(p_3) + \bar{\nu}(p_4))) + \gamma(p_5)$	NNLO
306	$Z(\rightarrow 3(\nu(p_3) + \bar{\nu}(p_4))) + \gamma(p_5) + \gamma(p_6)$	NLO + F
307	$Z(\rightarrow 3(\nu(p_3) + \bar{\nu}(p_4))) + \gamma(p_5) + f(p_6)$	NLO
308	$Z(\rightarrow 3(\nu(p_3) + \bar{\nu}(p_4))) + \gamma(p_5) + \gamma(p_6) + f(p_7)$	LO
309	$Z(\rightarrow 3(\nu(p_3) + \bar{\nu}(p_4))) + \gamma(p_5) + f(p_6) + f(p_7)$	LO
311	$f(p_1) + b(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + f(p_6)$	LO
316	$f(p_1) + b(p_2) \rightarrow W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + f(p_6)$	LO
321	$f(p_1) + c(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4)) + c(p_5) + f(p_6)$	LO
326	$f(p_1) + c(p_2) \rightarrow W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + c(p_5) + f(p_6)$	LO
331	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + c(p_5) + f(p_6)$ [c-s interaction]	LO
336	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + c(p_5) + f(p_6)$ [c-s interaction]	LO
341	$f(p_1) + b(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + f(p_6) [+f(p_7)]$	NLO
342	$f(p_1) + b(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + f(p_6) [+ \bar{b}(p_7)]$	(REAL)
346	$f(p_1) + b(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + f(p_6) + f(p_7)$	LO
347	$f(p_1) + b(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + b(p_5) + f(p_6) + \bar{b}(p_7)$	LO

351	$f(p_1) + c(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5) + f(p_6)[+f(p_7)]$	NLO
352	$f(p_1) + c(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5) + f(p_6)[+\bar{c}(p_7)]$	(REAL)
356	$f(p_1) + c(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5) + f(p_6) + f(p_7)$	LO
357	$f(p_1) + c(p_2) \rightarrow Z(\rightarrow e^-(p_3) + e^+(p_4)) + c(p_5) + f(p_6) + \bar{c}(p_7)$	LO
361	$c(p_1) + \bar{s}(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4))[\text{mc}=0 \text{ in NLO}]$	NLO
362	$c(p_1) + \bar{s}(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4))[\text{massless corrections only}]$	NLO
363	$c(p_1) + \bar{s}(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4))[\text{massive charm in real}]$	NLO
370	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + \gamma(p_5) + \gamma(p_6)$	LO
371	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + \gamma(p_5) + \gamma(p_6)$	LO
401	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5)$ [1,2 or 3 jets, 4FNS]	NLO
402	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + (b + \bar{b})(p_5)$ [1 or 2 jets, 4FNS]	NLO
403	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6)$ [2 or 3 jets, 4FNS]	NLO
406	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5)$ [1,2 or 3 jets, 4FNS]	NLO
407	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + (b + \bar{b})(p_5)$ [1 or 2 jets, 4FNS]	NLO
408	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + \bar{b}(p_6)$ [2 or 3 jets, 4FNS]	NLO
411	$f(p_1) + b(p_2) \rightarrow W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + f(p_6)$ [5FNS]	NLO
416	$f(p_1) + b(p_2) \rightarrow W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + f(p_6)$ [5FNS]	NLO
421	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5)$ [1,2 or 3 jets, 4FNS+5FNS]	NLO
426	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5)$ [1,2 or 3 jets, 4FNS+5FNS]	NLO
431	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + b(p_5) + \bar{b}(p_6) + f(p_7)$ [massive]	LO
436	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + b(p_5) + \bar{b}(p_6) + f(p_7)$ [massive]	LO
500	$W^+(\rightarrow \nu(p_3) + e^+(p_4)) + t(p_5) + \bar{t}(p_6)$ [massive]	NLO
501	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8)) + W^+(\nu(p_9), \mu^+(p_{10}))$	NLO
502	(same as process 501 but with radiation in decay)	NLO
503	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + q(p_7) + q(p_8)) + W^+(\nu(p_9), \mu^+(p_{10}))$	NLO
506	$t(\rightarrow q(p_3) + q(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8)) + W^+(\nu(p_9), \mu^+(p_{10}))$	NLO
510	$W^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + t(p_5) + \bar{t}(p_6)$ [massive]	NLO
511	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8)) + W^-(\mu^-(p_9), \bar{\nu}(p_{10}))$	NLO
512	(same as process 511 but with radiation in decay)	NLO
513	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + q(p_7) + q(p_8)) + W^-(\mu^-(p_9), \bar{\nu}(p_{10}))$	NLO
516	$t(\rightarrow q(p_3) + q(p_4) + b(p_5)) + \bar{t}(\rightarrow b(p_6) + e^-(p_7) + \bar{\nu}(p_8)) + W^-(\mu^-(p_9), \bar{\nu}(p_{10}))$	NLO
529	$Z(\rightarrow e^-(p_3) + e^+(p_4)) + t(p_5) + \bar{t}(p_6)$	LO
530	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow e^-(p_7) + \bar{\nu}(p_8) + b(p_6)) + Z(e^-(p_9), e^+(p_{10}))$	LO
531	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow e^-(p_7) + \bar{\nu}(p_8) + b(p_6)) + Z(b(p_9), b(p_{10}))$	LO
532	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow q(p_7) + \bar{q}(p_8) + b(p_6)) + Z(e^-(p_9), e^+(p_{10}))$	LO
533	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow e^-(p_7) + \bar{\nu}(p_8) + b(p_6)) + Z(e^-(p_9), e^+(p_{10}))$	LO

540	$H(b(p_3) + \bar{b}(p_4)) + t(p_5) + q(p_6)$	NLO
541	$H(b(p_3) + \bar{b}(p_4)) + \bar{t}(p_5) + q(p_6)$	NLO
544	$H(b(p_3) + \bar{b}(p_4)) + t(\nu(p_5) + e^+(p_6) + b(p_7)) + q(p_9)$	NLO
547	$H(b(p_3) + \bar{b}(p_4)) + \bar{t}(e^-(p_5) + \bar{\nu}(p_6) + b(p_7)) + q(p_9)$	NLO
550	$H(\gamma(p_3) + \gamma(p_4)) + t(p_5) + q(p_6)$	NLO
551	$H(\gamma(p_3) + \gamma(p_4)) + \bar{t}(p_5) + q(p_6)$	NLO
554	$H(\gamma(p_3) + \gamma(p_4)) + t(\nu(p_5) + e^+(p_6) + b(p_7)) + q(p_9)$	NLO
557	$H(\gamma(p_3) + \gamma(p_4)) + \bar{t}(e^-(p_5) + \bar{\nu}(p_6) + b(p_7)) + q(p_9)$	NLO
560	$Z(e^-(p_3) + e^+(p_4)) + t(p_5) + q(p_6)$	NLO
561	$Z(e^-(p_3) + e^+(p_4)) + \bar{t}(p_5) + q(p_6)$	NLO
562	$Z(e^-(p_3) + e^+(p_4)) + t(p_5) + q(p_6) + f(p_7)$	LO
563	$Z(e^-(p_3) + e^+(p_4)) + \bar{t}(p_5) + q(p_6) + f(p_7)$	LO
564	$Z(e^-(p_3) + e^+(p_4)) + t(\rightarrow \nu(p_5) + e^+(p_6) + b(p_7)) + q(p_8)$	NLO
566	$Z(e^-(p_3) + e^+(p_4)) + t(\rightarrow \nu(p_5) + e^+(p_6) + b(p_7)) + q(p_8) + f(p_9)$	LO
567	$Z(e^-(p_3) + e^+(p_4)) + \bar{t}(\rightarrow e^-(p_5) + \bar{\nu}(p_6) + \bar{b}(p_7)) + q(p_8)$	NLO
569	$Z(e^-(p_3) + e^+(p_4)) + \bar{t}(\rightarrow e^-(p_5) + \bar{\nu}(p_6) + \bar{b}(p_7)) + q(p_8) + f(p_9)$	LO
601	$H(b(p_3) + \bar{b}(p_4)) + H(\tau^-(p_5) + \tau^+(p_6))$	LO
602	$H(b(p_3) + \bar{b}(p_4)) + H(\gamma(p_5) + \gamma(p_6))$	LO
640	$t(p_3) + \bar{t}(p_4) + H(p_5)$	LO
641	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7) + e^-(p_8) + \bar{b}(p_6)) + H(b(p_9) + \bar{b}(p_{10}))$	LO
644	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{q}(p_7) + q(p_8) + \bar{b}(p_6)) + H(b(p_9) + \bar{b}(p_{10}))$	LO
647	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7) + e^-(p_8) + \bar{b}(p_6)) + H(b(p_9) + \bar{b}(p_{10}))$	LO
651	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7) + e^-(p_8) + \bar{b}(p_6)) + H(\gamma(p_9) + \gamma(p_{10}))$	LO
654	$t(\rightarrow \nu(p_3) + e^+(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{q}(p_7) + q(p_8) + \bar{b}(p_6)) + H(\gamma(p_9) + \gamma(p_{10}))$	LO
657	$t(\rightarrow q(p_3) + \bar{q}(p_4) + b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7) + e^-(p_8) + \bar{b}(p_6)) + H(\gamma(p_9) + \gamma(p_{10}))$	LO
661	$t(\rightarrow \nu(p_3)e^+(p_4)b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7)e^-(p_8)\bar{b}(p_6)) + H(W^+(p_9, p_{10})W^-(p_{11}, p_{12}))$	LO
664	$t(\rightarrow \nu(p_3)e^+(p_4)b(p_5)) + \bar{t}(\rightarrow \bar{q}(p_7)q(p_8)\bar{b}(p_6)) + H(W^+(p_9, p_{10})W^-(p_{11}, p_{12}))$	LO
667	$t(\rightarrow q(p_3)\bar{q}(p_4)b(p_5)) + \bar{t}(\rightarrow \bar{\nu}(p_7)e^-(p_8)\bar{b}(p_6)) + H(W^+(p_9, p_{10})W^-(p_{11}, p_{12}))$	LO

Table 32: Processes indicated by choice of the variable `nproc`.

800	$V \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Vector Mediator]	NLO
801	$A \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Axial Vector Mediator]	NLO
802	$S \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Scalar Mediator]	NLO
803	$PS \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Pseudo Scalar Mediator]	NLO
804	$GG \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Gluonic DM operator]	NLO
805	$S - -(\chi(p_3) + \bar{\chi}(p_4)) + f(p_5)$ [Scalar Mediator, mt loops]	NLO
820	$V \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5)$ [Vector Mediator]	NLO + F
821	$A \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5)$ [Axial Vector Mediator]	NLO + F
822	$S \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5)$ [Scalar Mediator]	NLO + F
823	$PS \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5)$ [Pseudo Scalar Mediator]	NLO + F
840	$V \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5) + f(p_6)$ [Vector Mediator]	LO
841	$A \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5) + f(p_6)$ [Axial Vector Mediator]	LO
842	$S \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5) + f(p_6)$ [Scalar Mediator]	LO
843	$PS \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5) + f(p_6)$ [Pseudo Scalar Mediator]	LO
844	$GG \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + f(p_5) + f(p_6)$ [Gluonic DM operator]	LO
845	$V \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5) + f(p_6)$ [Vector Mediator]	LO
846	$A \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5) + f(p_6)$ [Axial Vector Mediator]	LO
847	$S \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5) + f(p_6)$ [Scalar Mediator]	LO
848	$PS \rightarrow (\chi(p_3) + \bar{\chi}(p_4)) + \gamma(p_5) + f(p_6)$ [Pseudo Scalar Mediator]	LO
902	Check of Volume of 2 particle phase space	
903	Check of Volume of 3 particle phase space	
904	Check of Volume of 4 particle phase space	
905	Check of Volume of 5 particle phase space	
906	Check of Volume of 6 particle phase space	
908	Check of Volume of 8 particle phase space	
909	Check of Volume of 4 particle massive phase space	
910	Check of Volume of 3 particle (2 massive) phase space	
911	Check of Volume of 5 particle W+t (with decay) massive phase space	
912	Check of Volume of 5 particle W+t (no decay) massive phase space	
913	Check of Volume of 5 particle W+t+g (in decay) massive phase space	
914	Check of Volume of 5 particle W+t+g (in production) massive phase space	

Table 33: Processes indicated by choice of the variable `nproc`.

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